

ACADEMIC
EXCELLENCE
PROGRAM

Exercise Sheet

Vectors – Chapters 05–14

WACE Topic: Specialist Mathematics Year 11 – Vectors

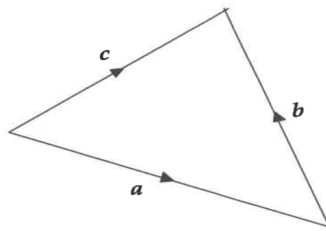
05 Addition & Subtraction of Vectors (Using Trigonometry)

Calculator Free

Question 1.

[2 marks: 1, 1]

Vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are as drawn in the accompanying diagram.



(a) Express $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Express $\mathbf{a}$ in terms of $\mathbf{b}$ and $\mathbf{c}$.

Calculator Assumed

Question 2.

[7 marks: 3, 4]

The angle between $\mathbf{u}$ and $\mathbf{u} - \mathbf{v}$ is 30° , $|\mathbf{v}| = 5$ and $|\mathbf{u} - \mathbf{v}| = 10$.

(a) Draw a clearly labelled sketch of the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.

- (b) Use the rules of trigonometry to find $|\mathbf{u}|$ and the angle between $\mathbf{u}$ and $\mathbf{v}$.

Question 3.*[6 marks: 2, 4]*

An aircraft is flying with a speed of 400 kmh^{-1} along bearing 145° . The aircraft is buffeted by a strong wind of magnitude 80 kmh^{-1} blowing from bearing 240° .

- (a) Draw a sketch to indicate the actual direction of the aircraft.

- (b) Find the ground speed and direction of the aircraft.

Question 4.*[6 marks: 3, 3]*

A boy intends to swim across a river of width 20 metres to the opposite bank. The river flows at a steady rate of 1 kmh^{-1} . The boy can swim at a steady speed of 2 kmh^{-1} .

- (a) In what direction should the boy be headed so that he ends up at the opposite bank directly opposite to where he started off?

- (b) Find the time taken for the swim in part (a).

06 Components & Position Vectors I

Calculator Free**Question 5.***[8 marks: 2, 1, 1, 4]*

Given that $\mathbf{a} = -2\mathbf{i} + 6\mathbf{j}$ and $\mathbf{b} = 5\mathbf{i} - 4\mathbf{j}$, find:

- (a) $|\mathbf{a} + \mathbf{b}|$.
- (b) the unit vector parallel to $\mathbf{a} + \mathbf{b}$.
- (c) a vector that is parallel to $\mathbf{a} + \mathbf{b}$ but with a magnitude of 5.
- (d) $\mathbf{a}$ in terms of $\mathbf{p}$ and $\mathbf{q}$ where $\mathbf{p} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{q} = -3\mathbf{i} + 2\mathbf{j}$.

Question 6.*[3 marks]*

The point K divides the line segment AB internally in the ratio 4 : 1. Use a vector method to find the position vector of K if $\mathbf{OB} = -\mathbf{i} + 2\mathbf{j}$ and $\mathbf{AB} = 15\mathbf{i} - 5\mathbf{j}$.

Calculator Assumed**Question 7.***[5 marks]*

Vector $\mathbf{u}$ has magnitude 100 kmh^{-1} and acts in the direction 040° . Vector $\mathbf{v}$ has magnitude 150 kmh^{-1} and acts in the direction 280° . Let $\mathbf{i}$ be the unit vector in the West–East direction and $\mathbf{j}$ be the unit vector in the South–North direction. Use vector components to find the magnitude and direction of $\mathbf{u} - 2\mathbf{v}$.

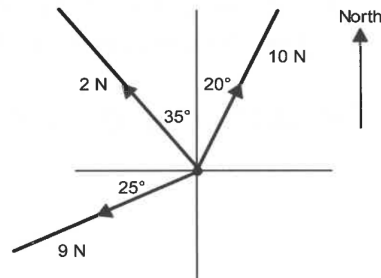
07 Components & Position Vectors II

Calculator Assumed

Question 8.

[6 marks: 3, 3]

The diagram below shows the forces acting on a body. The forces are all on the same plane.

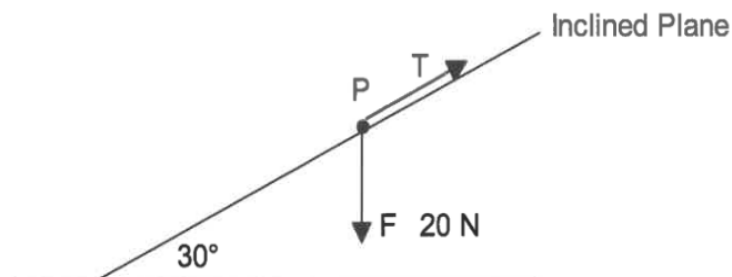


- (a) Find the magnitude of the resultant.
- (b) Find the magnitude and the direction of a single force that will keep this system in equilibrium.

Question 9.

[5 marks: 2, 2, 1]

In the diagram below, a particle P is on a plane inclined at an angle of 30° to the horizontal. A vertical force F of magnitude 20 N is acting on P as shown. Force T parallel to the inclined plane is applied to prevent P from slipping down the inclined plane.



- (a) Find the magnitude of the component of F parallel to the inclined plane.

(b) Find the magnitude of the component of F perpendicular to the inclined plane.

(c) Find the magnitude of force T .

Question 10.

[11 marks: 1, 1, 4, 2, 3]

A light plane can fly at 80 km per hour in still air. The pilot wishes to fly from O to a neighbouring airstrip Q , located 40 km from O in the direction 060° . A constant wind of 20 km per hour is blowing from the North. $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the Easterly and Northerly directions respectively.

(a) Write in terms of $\mathbf{i}$ and $\mathbf{j}$ the position vector of Q relative to O .

(b) Write in terms of $\mathbf{i}$ and $\mathbf{j}$ the velocity vector of the wind.

(c) Find the velocity vector the pilot should set so that the plane flies directly to Q .

(d) Find the resultant speed of the plane.

(e) Find the difference in flying time (to the nearest minute) caused by the wind.

08 Components & Position Vectors III

Calculator Assumed

Question 11.*[6 marks: 1, 1, 1, 3]*

A particle P, initially at $5\mathbf{i} - 10\mathbf{j}$ metres, moves with velocity $3\mathbf{i} + 4\mathbf{j} \text{ ms}^{-1}$.

- (a) Find the position vector of P after 10 seconds.

- (b) Find the distance travelled by P after 10 seconds.

- (c) Find the position vector of P after t seconds.

- (d) When is P at a point with position vector $(65\mathbf{i} + 70\mathbf{j})$ metres.

Question 12.*[9 marks: 3, 3, 3]*

The position vectors of particles A and B, t hours after 12 noon, are $\mathbf{r} = 12\mathbf{i} + 3\mathbf{j} + t(3\mathbf{i} + 4\mathbf{j})$ and $\mathbf{r} = -3\mathbf{i} - 5\mathbf{j} + t(2\mathbf{i} + 6\mathbf{j})$ metres respectively.

- (a) Find in terms of t , the distance between A and B t hours after 12 noon.

- (b) Find when A and B are 18 metres apart.

- (c) Find when A is closest to B and find this distance.

Question 13.

[12 marks: 2, 4, 2, 4]

At 9.00 am, boat A is located at $2\mathbf{i} + 3\mathbf{j}$ km and is travelling with velocity $2\mathbf{i} - 3\mathbf{j}$ km per hour. At 10.00 am, boat B is located at $2\mathbf{i} + 3\mathbf{j}$ km and is travelling with a speed of 5 km per hour. Let the velocity of boat B be $x\mathbf{i} + y\mathbf{j}$ km per hour.

- (a) Find the position vectors of A and B t hours after 10.00 am.
- (b) Show that when B intercepts A, $y = -1.5x$.
- (c) Hence, find the direction B should take in order to intercept A.
- (d) Find when B intercepts A.

09 Relative Displacement

Calculator Free**Question 14.**

[4 marks: 1, 1, 2]

The position vectors of the points A, B, and C with respect to the origin O are $\mathbf{i} + \mathbf{j}$, $2\mathbf{i} - \mathbf{j}$ and $-4\mathbf{i} + 5\mathbf{j}$ respectively.

- (a) Find the position vector of A relative to B.

- (b) Find the displacement of C relative to A.
- (c) The position vector of the point D relative to C is $-2\mathbf{i} - 3\mathbf{j}$. Find the position vector of D relative to O.

Calculator Assumed

Question 15.

[5 marks: 2, 1, 2]

The position vector of Peter relative to a flag pole is $20\mathbf{i} + 40\mathbf{j}$ metres. Relative to Peter, Joe has position vector $5\mathbf{i} - 15\mathbf{j}$ metres.

- (a) Find the position vector of Joe relative to the flagpole.
- (b) Hence, find the distance between Joe and the flag pole.
- (c) The position vector of Kelly relative to Joe is $a\mathbf{i} + 20\mathbf{j}$ metres. Find the value of a if the distance between Kelly and Joe is 50 metres.

10 Relative Velocity

Calculator Assumed

Question 16.

[7 marks: 1, 1, 3, 2]

Relative to an observer at O, A is moving with velocity $6\mathbf{i} + 9\mathbf{j} \text{ ms}^{-1}$ and B is moving with

velocity $-3\mathbf{i} + 4\mathbf{j} \text{ ms}^{-1}$.

(a) Find the velocity of A relative to B.

(b) What is the speed of A relative to B?

(c) The velocity of C relative to B is $4\mathbf{i} - 5\mathbf{j} \text{ ms}^{-1}$. Find the velocity of C relative to A.

(d) In what direction is C moving relative to A?

Question 17.

[7 marks: 3, 4]

L is travelling along bearing 060° with a speed of 10 kmh^{-1} . M is travelling along bearing 210° with a speed of 5 kmh^{-1} .

(a) Draw a clearly labelled vector diagram indicating the velocity vector of L relative to M.

(b) Use trigonometry to find the speed and direction of L relative to M.

11 Relative Vectors

Calculator Assumed

Question 18.*[6 marks: 2, 4]*

A yacht is sailing on a bearing of 050° with speed 12 kmh^{-1} . Sarah on the yacht measures the wind as blowing with a speed of 10 kmh^{-1} from a bearing of 300° .

- (a) Sketch a clearly labelled velocity vector diagram that shows the relationship between the velocity of the yacht, the true velocity of the wind and the velocity of wind relative to the yacht.

- (b) Find the true speed and direction of the wind.

Question 19.*[9 marks: 3, 3, 3]*

Tom is in a hot-air balloon travelling with velocity $3\mathbf{i} + 4\mathbf{j} \text{ kmh}^{-1}$ and to Tom the wind is blowing along bearing 045° . Jerry is in another hot-air balloon flying at the same altitude and travelling with velocity $-4\mathbf{i} + \mathbf{j}$. To Jerry, the wind is blowing along bearing 60° . Let the true velocity of the wind be $x\mathbf{i} + y\mathbf{j} \text{ kmh}^{-1}$. $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the Easterly and Northerly directions respectively.

- (a) Find in terms of x and y , the velocity of the wind relative to Tom. Hence, show that $x - y = -1$.

- (b) Find in terms of x and y , the velocity of the wind relative to Jerry. Hence, show that $x - \sqrt{3}y = -4 - \sqrt{3}$.

- (c) Find the true velocity of the wind.

Question 20.

[11 marks: 2, 2, 5, 2]

When the clock struck one, relative to a clock tower, the position vector of a cat is $6\mathbf{i} + 10\mathbf{j}$ m. The cat is running with velocity $2\mathbf{i} + 2\mathbf{j}$ ms^{-1} . At the same time, relative to the same clock tower, the position vector of a dog is $-4\mathbf{i} - 6\mathbf{j}$. The dog can run at 5 ms^{-1} .

- (a) Find the position vector of the cat relative to the dog.

Let the velocity vector of the dog for it to intercept the cat be $x\mathbf{i} + y\mathbf{j}$ ms^{-1} .

- (b) Find in terms of x and y , the velocity of the dog relative to the cat.
- (c) Use your answers in (a) and (b) to find x and y for the dog to intercept the cat. Assume that the cat continues running with the same speed and in the same direction.
- (d) When and where does the interception take place.

Question 21.

[16 marks: 1, 1, 3, 1, 4, 3, 3]

The locations and velocities of three boats at 0900 hours are given in the table below. Assume that each boat moves with constant velocity.

Boat	Location (km)	Velocity (kmh^{-1})
A	$20\mathbf{i} + 40\mathbf{j}$	$-4\mathbf{i} - 6\mathbf{j}$
B	$-25\mathbf{i} + 20\mathbf{j}$	$5\mathbf{i} - 2\mathbf{j}$
C	$35\mathbf{i} - 10\mathbf{j}$	$-7\mathbf{i} + 4\mathbf{j}$

- (a) Find the position vector of A relative to B at 0900 hours.

- (b) Find the velocity of B relative to A.
- (c) Use your answer in (b) to determine at what time B will meet A.
- (d) Determine the position vector of the point where A will meet B.
- (e) Find the distance between A and C t hours after 0900 hours.
- (f) Find the time when C will meet A.
- (g) Prove that all three boats will arrive at a similar spot at the same time.

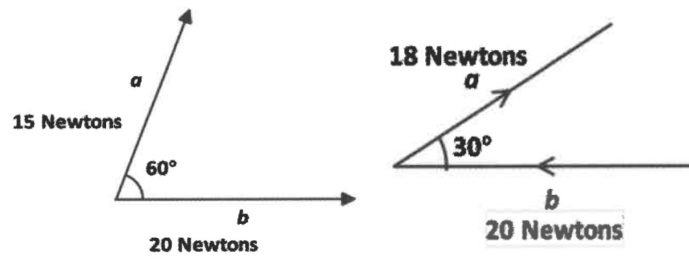
12 Scalar Product I

Calculator Free

Question 22.

[4 marks: 2, 2]

Find the scalar product between the vectors $\mathbf{a}$ and $\mathbf{b}$ as given:



(a) Left-hand diagram (15 N at 60° to 20 N).

(b) Right-hand diagram (18 N at 30° to 20 N, opposite sense).

Question 23.

[4 marks: 2, 1, 1]

Given that $|a| = 8$ and $|b| = 5$, and if $a \cdot b = 20\sqrt{2}$, find θ , the acute angle between:

(a) a and b

(b) $2a$ and $3b$

(c) a and $-b$.

Question 24.

[4 marks: 2, 2]

Given that p and q are perpendicular, prove that:

(a) $p \cdot q = 0$

(b) $(\mathbf{p} + \mathbf{q}) \cdot (\mathbf{p} + \mathbf{q}) = |\mathbf{p}|^2 + |\mathbf{q}|^2$

Calculator Assumed

Question 25.

[6 marks: 3, 3]

Given that $|\mathbf{u}| = 10$ and $(\mathbf{u} + 2\mathbf{v}) \cdot (\mathbf{u} + \mathbf{v}) = 408$.

(a) Find $|\mathbf{v}|$ if $\mathbf{u}$ and $\mathbf{v}$ are perpendicular.

(b) Find $|\mathbf{v}|$ if $\mathbf{u}$ and $\mathbf{v}$ are parallel and in opposite directions.

13 Scalar Product II

Calculator Free

Question 26.

[9 marks: 1, 2, 2, 2, 2]

The vectors $\mathbf{a}$ and $\mathbf{b}$ have magnitudes 3 and 4 respectively. The acute angle between $\mathbf{a}$ and $\mathbf{b}$ is $\cos^{-1} \frac{1}{3}$. Find in terms of the vectors $\mathbf{a}$ and/or $\mathbf{b}$:

(a) the scalar projection of $\mathbf{a}$ onto $\mathbf{b}$.

(b) the vector projection of $\mathbf{a}$ onto $\mathbf{b}$.

(c) the vector projection of $\mathbf{b}$ onto $\mathbf{a}$.

(d) $\mathbf{v}$, the component of $\mathbf{a}$ that is perpendicular to $\mathbf{b}$.

(e) the magnitude of $\mathbf{v}$, the component of $\mathbf{a}$ that is perpendicular to $\mathbf{b}$.

Question 27.

[4 marks: 2, 2]

Given $\mathbf{a} = \langle 5, 10 \rangle$ and $\mathbf{b} = \langle 4, 3 \rangle$.

(a) Find the component of $\mathbf{a}$ that is parallel to $\mathbf{b}$.

(b) Find the component of $\mathbf{a}$ that is perpendicular to $\mathbf{b}$.

Calculator Assumed

Question 28.

[7 marks: 5, 2]

The vector projection of $\mathbf{a}$ onto $\mathbf{b}$ is $\frac{1}{2}\mathbf{b}$. The vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\mathbf{a}$.

(a) Show mathematically that $|\mathbf{b}|^2 = 2|\mathbf{a}|^2$.

(b) Find the acute angle between $\mathbf{a}$ and $\mathbf{b}$.

14 Scalar Product III

Calculator Assumed

Question 29.*[6 marks: 1, 5]*

At 12 noon, A is at the point with position vector $3\mathbf{i} + 2\mathbf{j}$ km and moving with velocity $4\mathbf{i} - 2\mathbf{j}$ kmh⁻¹. B is a stationary object at the point with position vector $3\mathbf{i} - \mathbf{j}$ km.

- (a) Find the position vector of A at time t hours after 12 noon.

- (b) Use a scalar product method to find when A is closest to B. State this distance.

Question 30.*[10 marks: 2, 2, 1, 5]*

A force $\mathbf{F}_1$ of $\langle 4, 4 \rangle$ Newtons is applied to a body and causes the body to be displaced by $\langle 3, 1 \rangle$ metres.

- (a) Find the component of the applied force along the direction of motion.

- (b) Find the component of the applied force perpendicular to the direction of motion.

- (c) Determine the work done by the applied force.

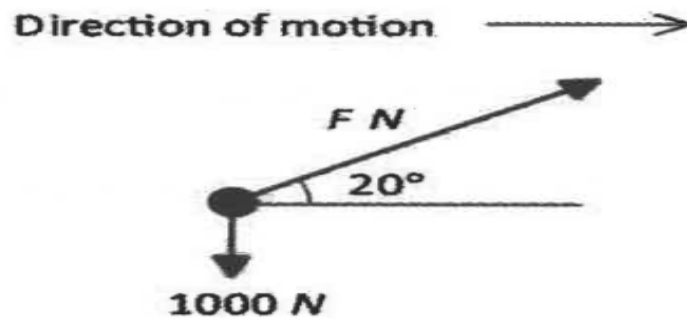
- (d) Another force $\mathbf{F}_2$ of $\langle x, y \rangle$ Newtons is applied to the same body. But half as much work is required to cause the same displacement to the body.
 - (i) Find a possible pair of values for x and y .

- (ii) Find x and y if $|\mathbf{F}_2| = 2\sqrt{2}$ Newtons.

Question 31.

[6 marks: 3, 2, 1]

An object of weight 1000 Newtons is being pulled along a horizontal surface by a force of magnitude F Newtons inclined at an angle of 20° to the surface. The motion of the object is opposed by a horizontal force of magnitude 500 N.

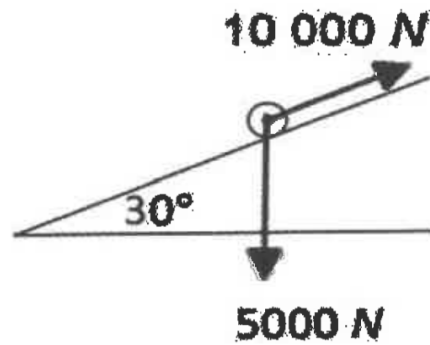


- (a) Find in terms of F , the magnitude of the pulling force perpendicular to the surface. Hence, find F .
- (b) Find the component of the pulling force in the direction of motion. Hence, find the magnitude of the resultant force in the direction of motion.
- (c) Find the work done by the pulling force in moving the object 50 m in its direction of motion.

Question 32.

[7 marks: 2, 1, 2, 2]

A body of weight 5000 Newtons is pulled up along a plane inclined at an angle of 30° with the horizontal by a force of magnitude 10000 N. There is a force of magnitude 500 N opposing the motion of the body.



- (a) Find the component of the gravitational force on the body along the inclined plane. Hence, find the magnitude of the resultant force in the direction of the motion of the body.
- (b) Find the work done by the resultant force when the body has moved 10 m along the inclined plane.
- (c) Find the work done by the resultant force when the body has ascended a vertical distance of 10 m.
- (d) The work done by the resultant force in moving the body up the inclined plane so that the change in its horizontal position is x metres is 280 kJ. Find x .