

Exercise Sheet Chapter 16a Combinatorics

Date: Wednesday 22 July – Thursday 23 July

WACE Topic: Specialist Year 11 – Combinatorics

01 Combinatorics I

Calculator Free

Question 1

[9 marks: 2, 2, 2, 3]

Find k if:

(a) $\frac{10!}{k!} = 720$

(b) $\frac{k!}{8!} = 990$

(c) ${}^kP_3 = 210$

(d) $\frac{10! - k!}{k!} = 30\,239$

Question 2

[4 marks: 1, 1, 1, 1]

The letters of the word RICHES are rearranged in a line. No letter is used more than once. Write mathematical expressions for:

- (a) the total number of possible arrangements.

- (b) the number of arrangements with the letters I and E adjacent.
- (c) the number of arrangements with the letters I and E not adjacent.
- (d) the number of arrangements where the vowels are adjacent and the letters C and H are adjacent.

Question 3*[4 marks: 1, 1, 1, 1]*

Codes are formed by using the letters of the word MATHS, with no letter being used more than once:

- (a) How many five letter codes start with M?
- (b) How many five letter codes have the letters A and M together?
- (c) How many four letter codes start with the letter M?
- (d) How many three letter codes end with the letter S.

Calculator Assumed**Question 4***[4 marks: 1, 1, 1, 1]*

Three year 11 students and four year 12 students are to be arranged in a line.

- (a) In how many ways can this be done?

- (b) In how many arrangements are all the year 11 students next to each other?

- (c) How many arrangements have all students of the same year group adjacent to each other?

- (d) How many arrangements have no student of the same year group adjacent to each other?

Question 5*[9 marks: 1, 2, 3, 3]*

Six digit numbers are formed using the digits 1, 2, 3, 4, 5, and 6. Each digit is used only once.

- (a) How many six digit numbers are possible?

- (b) How many six digit even numbers are possible?

- (c) How many five digit even numbers greater than 40 000 are possible?

- (d) How many even numbers greater than 40 000 are possible?

02 Combinatorics II

Calculator Assumed

Question 6

[4 marks: 1, 1, 2]

In a group of 40 students, there were 10 boys who were colour vision deficient (CVD) and 15 girls who were not CVD. There were as many boys who were not CVD as there were boys who were CVD. How many of these students:

- (a) were boys?
- (b) were either boys or colour vision deficient?
- (c) were colour vision deficient?

Question 7

[7 marks: 1, 1, 1, 1, 2, 1]

Flags of 10 different nations including that of Australia and New Zealand are to be flown from 10 flag poles set in a line. The flag poles are labelled poles 1 to 10. How many ways are there of assigning a flag to each of these poles if the:

- (a) Australian flag must be flown from pole 1?
- (b) Australian flag or New Zealand flag must be flown from pole 1?
- (c) Australian flag and the New Zealand flag must be flown from poles 1 and 10 respectively?

- (d) Australian flag must be flown from pole 1 and the New Zealand flag must not to be flown from pole 10.
- (e) Australian flag must be flown from pole 1 or the New Zealand flag must be flown from pole 10.
- (f) Australian flag must be flown from pole 1 or the New Zealand flag must be flown from pole 10 but not both at the same time.

Question 8*[7 marks: 2, 2, 3]*

Ten potted plants including four pots of roses of different shades of red and three pots of azaleas (each of a different colour) are to be arranged in a line along a footpath. How many arrangements will have:

- (a) the potted roses adjacent to each other?
- (b) the potted azaleas adjacent to each other?
- (c) the roses adjacent to each other or the azaleas adjacent to each other?

Question 9*[6 marks: 2, 2, 2]*

An analysis was conducted on the tickets bought by 150 patrons of the Perth International Festival.

The survey was restricted to the three categories: Classical Music, Contemporary Music, Theatre, Circus & Dance (TCD).

- All had bought tickets to at least one of these categories.
 - 30 had tickets to all categories.
 - A total of 100 had tickets to a Classical Music event.
 - A total of 90 had tickets to a Contemporary Music event.
 - 80 had tickets to a Classical Music event as well as a TCD event.
 - 50 had tickets to a Classical Music as well as a Contemporary Music event.
 - 30 had tickets to a Contemporary Music as well as a TCD event.
- (a) How many patrons had tickets to a Classical Music and a Contemporary Music event but not a Theatre, Circus & Dance event?
- (b) How many patrons had tickets to only a Classical Music event?
- (c) How many patrons had tickets to a Theatre, Circus & Dance event?

Question 10

[6 marks: 3, 3]

Consider the set of integers between 1 000 and 5 000 inclusive.

- (a) Complete the following table listing the number of multiples of n within this set for the given values of n .

n	Number of multiples of n in this set.
2	
3	
5	
6	
10	
15	
30	

- (b) Find the number of integers in this set that are multiples of 2, 3 or 5.

03 Combinatorics III

Calculator Free

Question 11

[8 marks: 2, 2, 2, 2]

(a) Determine the $\binom{9}{6}$.

(b) Evaluate $\binom{9}{3} + \binom{9}{4}$.

(c) Simplify ${}^{10}C_5 \times {}^5P_5$. You are not required to evaluate your answer.

(d) Given that ${}^5P_{r+1} = {}^5P_r$, find r .

Question 12

[13 marks: 2, 2, 2, 4, 3]

(a) Find n and r if ${}^nC_r = \frac{n \times (n-1) \times 98}{3 \times 2 \times 1}$.

(b) Find n and r if ${}^nP_r = 20 \times 19 \times 18 \times 17$

(c) Find a and b if ${}^{30}C_a = {}^{3a}C_b$.

(d) Find all possible values of a and b if ${}^{12}C_a = {}^{12}C_{2a+b}$.

(e) Find a possible set of values for a and b if $10 \times {}^9P_4 = 6 \times {}^aP_b$.

Calculator Assumed

Question 13

[12 marks: 1, 3, 2, 3, 3]

A committee of 9 people are to be selected from 10 Labor, 8 Liberal and 5 Green politicians. How many different ways can the committee be selected if:

(a) there are no restrictions

(b) all three political parties are equally represented

(c) there are no Green representatives

(d) the Liberal representatives are in the (absolute) majority

(e) a husband and wife pair, Alex and Alice, cannot be in the same committee.

Question 14

[10 marks: 1, 2, 3, 4]

[TISC]

Wei has a collection of 20 stickers in her pink box and 25 stickers in her blue box. All these stickers are different from each other.

- (a) In how many ways can Wei pick 3 stickers from her pink box?

- (b) In how many ways can Wei pick 2 stickers from her blue box and arrange them in a line?

- (c) In how many ways can Wei pick 3 stickers from her pink box and 2 stickers from her blue box and arrange them in a line if:
 - (i) there are no restrictions as to how the stickers are arranged?

 - (ii) all the stickers from the blue box must be together?

Calculator Free**Question 15**

[12 marks: 1, 2, 2, 2, 2, 3]

Consider the letters of the word CONQUEST. Write mathematical expressions for the number of ways the letters of this word can be *rearranged*:

- (a) if the first letter must be a consonant.

- (b) if the first three letters must be consonants.

- (c) if one of the first three letters must be a consonant.

- (d) if at least one of the first three letters must be a consonant.

- (e) if the vowels are to be sandwiched between two consonants (that is, a vowel must be preceded by a consonant and this vowel must also be followed by a consonant).

Question 16

[6 marks: 3, 3]

A number is divisible by four if the last two digits of the number is divisible by four. For example, 4564 is divisible by four because the last two digits “64” is divisible by four; but 4502 is not divisible by four because the last two digits “02” is not divisible by four. Using the digits 0 to 9 inclusive, write an expression for the number of five digit numbers divisible by four that can be formed:

(a) if digits may be repeated?

(b) if no digit is to be repeated?

Calculator Assumed**Question 17**

[8 marks: 3, 2, 3]

A password consisting of four characters is to be chosen from 10 digits, 26 upper case letters, 26 lower case letters and a set of 32 symbols.

(a) Complete the table below.

Composition of password	Number of different passwords
2 lower case letters	
2 symbols	
2 lower case letters and 2 symbols	

(b) Determine the number of possible passwords with either two lower case letters or two symbols.

(c) Determine the number of possible passwords with either two lower case letters or two upper case letters.