



# Exercise Sheet – Essential Practice Sets, Combinatorics, Probability I & II

WACE Topic: Mathematics Methods Year 11

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## 01 Sets

### *Calculator Assumed*

#### Question 1

[6 marks: 1, 1, 2, 2]

Given that  $\mathbf{U} = \{x \mid 50 \leq x \leq 70, x \text{ is an integer}\}$ ,

$\mathbf{A} = \{51, 53, 65, 68\}$ ,  $\mathbf{B} = \{62, 64, 65, 66\}$  and  $\mathbf{C} = \{51, 53, 66, 70\}$ .

(a) Find  $|\mathbf{U}|$ .

(b) Find  $\mathbf{A} \cup \mathbf{B}$ .

(c) Find  $n(\mathbf{C} \cap \overline{\mathbf{B}})$ .

(d) Find  $|\overline{\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}}|$ .

**Question 2**

[8 marks: 2, 2, 2, 2]

Given that  $\mathbf{U} = \{x \mid 1 \leq x \leq 20, x \text{ is an integer}\}$ ,  $\mathbf{A} = \{x \mid x \text{ is a prime number}\}$ ,  $\mathbf{B} = \{x \mid x \text{ is a square number}\}$  and  $\mathbf{C} = \{x \mid x \text{ is a multiple of 3}\}$ .

(a) Find  $\mathbf{B} \cap \mathbf{C}$ .

(b) Find  $\mathbf{A} \cup \mathbf{B}$ .

(c) Find  $|\mathbf{A} \cup (\mathbf{B} \cap \mathbf{C})|$ .

(d) Find  $|(\mathbf{A} \cup \mathbf{B}) \cap \mathbf{C}|$ .

## 02 Combinatorics

### *Calculator Free*

#### Question 1

[7 marks: 1, 2, 2, 2]

Determine the value of integer  $r$  (where  $r \geq 0$ ) in each of the following equations:

(a)  $\binom{12}{8} = \binom{12}{r}$

(b)  $\binom{30}{r} = \binom{30}{r+4}$

(c)  $\binom{25}{2r} - \binom{25}{r-2} = 0$

(d)  $\binom{r}{0} + \binom{r}{1} + \binom{r}{2} + \binom{r}{3} + \binom{r}{4} = 2^r$

**Question 2***[10 marks: 1, 2, 2, 3, 2]*

A committee of 9 people is to be selected from 10 Labor, 8 Liberal and 5 Green politicians. Write mathematical expressions for the number of different ways the committee can be selected if:

(a) there are no restrictions.

(b) all three political parties are equally represented.

(c) there are no Greens.

(d) the Liberal representatives are in the majority.

(e) the Labor husband and wife pair, Alex and Alice, cannot be in the same committee.

## 03 Probability I

### *Calculator Assumed*

#### Question 1

[10 marks: 4, 6]

Allie's Café offers the following menu:

**Appetisers:** *Prawn Cocktail or Bruschetta*

**Main Meal:** *Fish or Chicken or Steak or Lamb*

**Dessert:** *Pavlova or Cheesecake or Apple Pie with Ice Cream*

Jenny orders an appetiser, one item for the main meal and one dessert. Jenny will not have Steak with Pavlova and must have Prawn Cocktail with Fish and Fish only with Prawn Cocktail.

- (a) Display Jenny's possible appetiser/main meal/dessert combinations in a clearly labelled tree diagram.
  
  
  
  
  
  
  
  
  
  
- (b) If each of Jenny's possible appetiser/main meal/dessert combinations are equally likely, find the probability that Jenny:
  - (i) chose chicken as a main meal.
  
  
  
  - (ii) did not choose cheesecake as a dessert.
  
  
  
  - (iii) chose cheesecake given that she chose chicken.
  
  
  
  - (iv) chose chicken given that she chose cheesecake.

**Question 2***[9 marks: 1, 2, 3, 3]**[TISC]*

Zico practices kicking a soccer ball from the penalty spot. From previous practices, on average, he scores 70 goals from 100 attempts.

- (a) Find the probability that Zico's first two kicks do not score goals.
  
- (b) Find the probability that Zico's first kick scores a goal but the next two kicks do not score goals.

If Zico has 10 kicks of the ball from the penalty spot, find the probability that,

- (c) he scores exactly 5 goals.
  
- (d) he scores goals only on the first, fifth, seventh and ninth kick.

**Question 3**

[11 marks: 2, 3, 3, 3]

The *Collett Boat Company* has a fleet of three boats. From company records for the last two years, the *Jupiter* is chosen by 55% of customers, the *Venus* by 28% of customers and the *Mars* by the remaining customers. The probabilities that each boat breaks down during a two-hour trip are *Jupiter* 0.2; *Venus* 0.15; *Mars* 0.3.

- (a) If all three boats are out on hire for a two-hour trip, find the probability that:
- (i) none breaks down.
  - (ii) the *Mars* and one other boat in this fleet breaks down.
- (b) Only one boat is out on hire for a two-hour trip. What is the probability that it will break down.
- (c) News comes through that the one boat out on hire has broken down. What is the probability that it is the *Jupiter*?

## 04 Probability II

### *Calculator Free*

#### Question 1

[8 marks: 4, 2, 2]

Given that  $\Pr(A') = \frac{3}{10}$ ,  $\Pr(B \mid A) = \frac{2}{7}$  and  $\Pr(A \mid B) = \frac{1}{2}$ :

(a) find  $\Pr(B)$ .

(b) find  $\Pr(A' \cap B')$ .

(c) determine with reasons if  $A$  and  $B$  are independent.

***Calculator Assumed*****Question 2***[10 marks: 1, 2, 4, 3]*

Given that  $\Pr(A) = p + 0.2$  and  $\Pr(B) = p + 0.3$  and  $\Pr(A \cap B) = p$ , calculate the value of  $p$  if:

(a)  $A$  and  $B$  are mutually exclusive events.

(b)  $\Pr(A \cup B) = 0.6$ .

(c)  $A$  and  $B$  are independent events.

(d)  $\Pr(A \mid B) = 0.4$ .

**Question 3**

[7 marks: 1, 2, 2, 2]

[TISC]

John drives to work each weekday morning. The route he takes passes through a set of traffic lights where he either has to stop at the lights or move through without stopping.

- If he has to stop at the lights, the probability that he will be late for work is 0.7.
- If he does not have to stop at the lights, the probability that he will be late for work is 0.2.

Overall, the probability that John will be late for work is 0.25.

- (a) Find the probability that he will not be late for work if he did not have to stop at the traffic lights.
- (b) Find the probability that John has to stop at the traffic lights.
- (c) Determine with reasons if John being late is independent of whether he has to stop at the lights.
- (d) Find the probability that John had to stop at the lights given that he was late for work.