19 Sets

Calculator Assumed

1. [6 marks: 1, 1, 2, 2]

   Given that $\mathbf{U} = \{ x \mid 50 \le x \le 70, x \text{ is an integer}\}$,
   $\mathbf{A} = \{51, 53, 65, 68\}$, $\mathbf{B} = \{62, 64, 65, 66\}$ and $\mathbf{C} = \{51, 53, 66, 70\}$.

   (a) Find $|\mathbf{U}|$.

   (b) Find $\mathbf{A} \cup \mathbf{B}$.

   (c) Find $n(\mathbf{C} \cap \overline{\mathbf{B}})$.

   (d) Find $|\overline{\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}}|$.

---

2. [7 marks: 1, 2, 2, 2]

   Given that $\mathbf{U} = \{ x \mid 0 \le x \le 20, x \text{ is an integer}\}$,
   $\mathbf{P} = \{ x \mid 5 \le x \le 14\}$, $\mathbf{Q} = \{ x \mid 3 \le x \le 9\}$ and $\mathbf{R} = \{2, 4, 6\}$.

   (a) Is $2 \in \mathbf{R}$?

   (b) Is $\mathbf{Q} \subset \mathbf{P}$? Justify your answer.

   (c) Find $n(\mathbf{Q}')$.

   (d) Find $|\mathbf{U} \cap \mathbf{P}|$.

Calculator Assumed

3. [6 marks: 1, 1, 2, 2]

Given that $\mathbf{U} = \{ x \mid -10 \le x \le 10, x \text{ is an integer} \}$,
$\mathbf{A} = \{ x \mid -10 \le x \le -1 \}$, $\mathbf{B} = \{ x \mid 0 \le x \le 10 \}$ and $\mathbf{C} = \{2, 3, 5, 7\}$.

(a) Find $\mathbf{A} \cap \mathbf{B}$.

(b) Find $\mathbf{A} \cup \mathbf{B}$.

(c) Find $n(\mathbf{B} \cap \overline{\mathbf{A}})$.

(d) Find $|(\mathbf{A} \cup \mathbf{B}) \cap \overline{\mathbf{C}}|$.

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4. [8 marks: 2, 2, 2, 2]

Given that $\mathbf{U} = \{ x \mid 1 \le x \le 20, x \text{ is an integer} \}$, $\mathbf{A} = \{ x \mid x \text{ is a prime number} \}$,
$\mathbf{B} = \{ x \mid x \text{ is a square number} \}$ and $\mathbf{C} = \{ x \mid x \text{ is a multiple of 3} \}$.

(a) Find $\mathbf{B} \cap \mathbf{C}$.

(b) Find $\mathbf{A} \cup \mathbf{B}$.

(c) Find $|\mathbf{A} \cup (\mathbf{B} \cap \mathbf{C})|$.

(d) Find $|(\mathbf{A} \cup \mathbf{B}) \cap \mathbf{C})|$.

Calculator Assumed

5. [9 marks: 2, 2, 3, 2]

Given that $\mathbf{U} = \{ x \mid 0 \le x \le 20, x \text{ is an integer}\}$, $\mathbf{A} = \{x \mid x \text{ is a factor of } 24\}$, $\mathbf{B} = \{x \mid x \text{ is a prime number}\}$ and $\mathbf{C} = \{x \mid x \text{ is a triangular number}\}$.

(a) Find $\mathbf{B} \cap \mathbf{C}$

(b) Find $\mathbf{C} \cup \mathbf{B}$

(c) $n(\mathbf{A} \cap \overline{\mathbf{B}})$

(d) An element is chosen at random from **U**. Find the probability that this element is from set **B**, given that it is from set **C**.

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6. [5 marks: 1, 1, 1, 2]

Given that $\mathbf{A} = \{1, 2, 3\}$, $\mathbf{B} = \{0, 1, 2\}$ and $\mathbf{C} = \{ (x, y) \mid x \in A, y \in B \}$.

(a) Find $\mathbf{A} \cap \mathbf{B}$

(b) Find $\mathbf{A} \cup \mathbf{B}$

(c) Is $(1, 2) \in C$?

(d) $|\mathbf{C}|$

20 Combinations

Calculator Free

1. [10 marks: 1, 2, 2, 2, 3]

   Evaluate each of the following:

   (a) 5!

   (b) $\binom{10}{5}$

   (c) $\binom{10}{5} \times 5!$

   (d) $\binom{40}{2}$

   (e) $\binom{60}{58}$

Calculator Free

2. [7 marks: 1, 2, 2, 2]

Determine the value of integer r (where $r \geq 0$) in each of the following equations:

(a) $\binom{12}{8} = \binom{12}{r}$

(b) $\binom{30}{r} = \binom{30}{r+4}$

(c) $\binom{25}{2r} - \binom{25}{r-2} = 0$

(d) $\binom{r}{0} + \binom{r}{1} + \binom{r}{2} + \binom{r}{3} + \binom{r}{4} = 2^r$

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3. [4 marks: 1, 3]

Expand completely each of the following:

(a) $(1 + x)^5$

(b) $(2 - x)^4$

Calculator Free

4. [13 marks: 1, 4, 5, 3]

Consider the expansion for $\left(x^2 - \frac{2}{x}\right)^{12}$ in *descending* powers of x.

(a) How many terms are there in this expansion?

(b) Find the third term in this expansion.

(c) Find a mathematical expression for the coefficient of the term in $\frac{1}{x^{12}}$.

(d) Find a mathematical expression for the term independent of x.

Calculator Free

5. [5 marks: 1, 2, 2]

Amy has a collection of 18 fluoro pens in her pink box and 24 fluoro pens in her blue box. Write mathematical expressions for the number of ways Amy can pick:
(a) three pens from her pink box.

(b) three pens from the pink box and four pens from the blue box.

(c) a dozen pens from both boxes.

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6. [10 marks: 1, 2, 2, 3, 2]

A committee of 9 people is to be selected from 10 Labor, 8 Liberal and 5 Green politicians. Write mathematical expressions for the number of different ways the committee can be selected if:
(a) there are no restrictions.

(b) all three political parties are equally represented.

(c) there are no Greens.

(d) the Liberal representatives are in the majority.

(e) the Labor husband and wife pair, Alex and Alice, cannot be in the same committee.

Calculator Assumed

7. [19 marks: 1, 3, 3, 4, 4, 4]

Consider the digits 0 to 9 inclusive and all the letters of the alphabet. Ten characters consisting of digits and letters are chosen. Determine the number of ways of choosing:

(a) all the even numbers and all the vowels.

(b) any six digits and any four letters.

(c) exactly four vowels.

(d) at least four odd digits.

(e) four vowels and four odd digits.

(f) four vowels or four odd digits.

21 Probability I

Calculator Free

1. [9 marks: 1, 2, 2, 2, 2]

   Let the universal set U be the set of all integers between 1 and 20 inclusive.
   Let M: $\{x \mid x \text{ is a multiple of } 6\}$ and F: $\{x \mid x \text{ is a factor of } 72\}$.

   (a) List all the elements in F.

   Use a Venn Diagram or a two-way table to the answer the following questions.

   (b) How many multiples of 6 are not factors of 72?

   (c) How many integers are either multiples of 6 or are factors of 72?

   (d) Find the probability that a randomly chosen integer:
   (i) is a neither a multiple of 6 nor a factor of 72.

   (ii) is a multiple of 6 given that it is a factor of 72.

Calculator Assumed

2. [8 marks: 2, 2, 2, 2]

The Mathematics department at a school conducted a random survey involving 200 students.

- 38 students did not have any calculator (scientific or CAS) with them
- 142 students had a CAS calculator with them
- 52 students had a scientific calculator with them

Use a Venn Diagram or a two-way table to the answer the following questions.

(a) Find the probability that a student chosen at random had only a CAS calculator.

(b) Find the probability that a student chosen at random had both a CAS calculator as well as a scientific calculator.

(c) Find the probability that a student chosen at random had either a CAS calculator or a scientific calculator.

(d) Find the probability that a student selected from those who had at least one type of calculator had both types of calculator.

Calculator Assumed

3. [8 marks: 2, 2, 2, 2]

In a group of 40 students, there are 10 boys who are colour vision impaired (CVA) and 15 girls who are not colour vision impaired. There are as many boys who are not colour vision impaired as there are boys who are colour vision impaired.

Use a Venn Diagram or a two-way table to the answer the following questions.

(a) How many girls were colour vision impaired?

(b) A student is randomly chosen from this group. Find the probability that this student is a girl.

(c) A student is randomly chosen from this group. Find the probability that this student is either a girl or is colour vision impaired.

(d) A student is randomly chosen from this group. Given that this student is either a girl or is colour vision impaired, find the probability that this student is colour vision impaired.

Calculator Assumed

4. [10 marks: 4; 1, 1, 2, 2]

Box A contains two red balls and a white ball. Box B contains one white ball and a red ball. A coin is tossed. If a tail appears, 2 balls are drawn without replacement from box B. If a head appears, 2 balls are drawn without replacement from box A.

(a) Use a tree diagram to display all the possible outcomes.

(b) If each outcome is equally likely, find the probability that:

(i) exactly two red balls are chosen.

(ii) exactly one white ball is chosen.

(iii) exactly two red balls are chosen given that box A was chosen.

(iv) box A was chosen given that exactly one white ball was chosen.

Calculator Assumed

5. [10 marks: 4, 6]

Allie's Café offers the following menu:

Appetisers: *Prawn Cocktail or Bruschetta*
Main Meal: *Fish or Chicken or Steak or Lamb*
Dessert: *Pavlova or Cheesecake or Apple Pie with Ice Cream*

Jenny orders an appetiser, one item for the main meal and one dessert. Jenny will not have Steak with Pavlova and must have Prawn Cocktail with Fish and Fish only with Prawn Cocktail.

(a) Display Jenny's possible appetiser/main meal/dessert combinations in a clearly labelled tree diagram.

(b) If each of Jenny's possible appetiser/main meal/dessert combination are equally likely, find the probability that Jenny:

(i) chose chicken as a main meal.

(ii) did not choose cheesecake as a dessert.

(iii) chose cheesecake given that she chose chicken.

(iv) chose chicken given that she chose cheesecake.

Calculator Assumed

6. [10 marks: 1, 3, 3, 3]

A red box has four books and a blue box has eight books. All books are different. A total of five books are chosen from these two boxes.

(a) In how many ways can this be done?

(b) What is the probability that all the books from the red box are chosen?

(c) What is the probability that at least one of the books chosen is from the red box?

(d) What is the probability that more books from the red box are chosen?

Calculator Assumed

7. [15 marks: 3, 3, 3, 3, 3]

Consider the digits 0 to 9 inclusive and all the letters of the English Roman alphabet. Twelve characters consisting of digits and letters are chosen.

(a) What is the probability that all the characters chosen are letters?

(b) What is the probability that all the digits are chosen?

(c) Given that all the characters chosen are letters, what is the probability that all the vowels are chosen?

(d) What is the probability that no vowels or even digits were chosen?

(e) What is the probability that at least one vowel or even digit was chosen?

Calculator Assumed

8. [11 marks: 1, 1, 1, 2, 3, 3]

Last year, Malcolm was late to school on average, 5 days out of 100 days.
Write mathematical expressions (but do not evaluate) for the probability that in a school week of 5 days, Malcolm is:

(a) late only on the first day.

(b) late on the first three days.

(c) late only on the first three days.

(d) late only on exactly three days.

(e) late on at least three days.

(f) late only the first and the fifth day given that he was late on exactly two days in the school week.

Calculator Assumed

9. [9 marks: 1, 2, 3, 3] [TISC]

Zico practices kicking a soccer ball from the penalty spot. From previous practices, on average, he scores 70 goals from 100 attempts.

(a) Find the probability that Zico's first two kicks do not score goals.

(b) Find the probability that Zico's first kick scores a goal but the next two kicks do not score goals.

If Zico has 10 kicks of the ball from the penalty spot, find the probability that,

(c) he scores exactly 5 goals.

(d) he scores goals only on the first, fifth, seventh and ninth kick.

Calculator Assumed

10. [11 marks: 2, 3, 3, 3]

The *Collett Boat Company* has a fleet of three boats. From Company records for the last two years, the *Jupiter* is chosen by 55% of customers, the *Venus* by 28% of customers and the *Mars* by the remaining customers. The probabilities that each boat breaks down during a two-hour trip are *Jupiter* 0.2; *Venus* 0.15; *Mars* 0.3.

(a) If all three boats are out on hire for a two-hour trip, find the probability that:
(i) none breaks down.

(ii) the *Mars* and one other boat in this fleet breaks down.

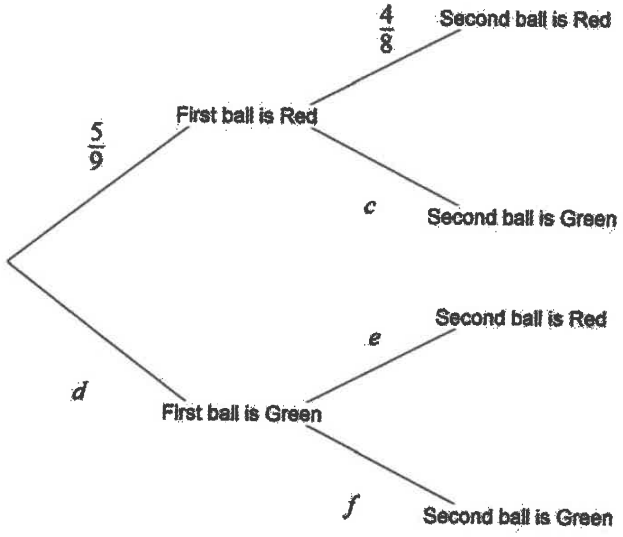
(b) Only one boat is out on hire for a two-hour trip. What is the probability that it will break down.

(c) News comes through that the one boat out on hire has broken down. What is the probability that it is the *Jupiter*?

Calculator Assumed

11. [9 marks: 4, 2, 3]

A box has five red balls and four green balls. Two balls are drawn without replacement from this box. The tree diagram indicates the associated outcomes and the corresponding probabilities.



(a) State the probability values c, d, e and f.

(b) Find the probability that both balls are red.

(c) Find the probability that both balls are of the same colour.
Show clearly how you obtained your answer.

Calculator Assumed

12. [10 marks: 2, 4, 4] [TISC]

Chin either drives to work or takes a train to work. The probability that he is on time for work is 0.86. The probability that he is late for work given that he drives to work is 0.3. The probability that he is on time for work given that he takes a train is 0.9.

(a) Find the probability that he is on time for work given that he drives to work.

(b) Find the probability that he drives to work.

(c) Given that he is late for work, what is the probability that he took the train to work.

Calculator Assumed

13. [9 marks: 1, 5, 3]

At a certain airport, the probability that a plane takes off on time given that weather conditions are fine is 0.9. The probability that a plane takes off on time given that weather conditions are bad is 0.7. The probability of weather conditions being fine or the plane taking off on time is 0.955.

(a) Find the probability that a plane does not take off on time given that weather conditions were bad.

(b) Find the probability of a plane taking off on time in fine weather conditions.

(c) Find the probability of weather conditions being fine given that a plane took off on time.

Calculator Assumed

14. [11 marks: 5, 6]

To be awarded a pass for a course, students are required to first pass a practical examination and then a theory examination. A student is only allowed one failure. A student that fails two examinations automatically fails the course. In all cases, a student is allowed to repeat a failed examination only once. The probability that a student will pass the practical examination on the first attempt is 0.8. The probability that a student will pass the theory examination given that the student has passed the practical examination is 0.9, even if the student failed the practical examination in the first attempt. The probability that a student fails outright (fails the practical examination on the first and second attempt) is 0.01 and the probability that a student fails the course is 0.045.

(a) Find the probability that a randomly chosen student fails the entire course given that the student failed the practical examination on the first attempt.

Calculator Assumed

14. (b) Find the probability that a randomly chosen student passes the course given that the student had to repeat an examination.

22 Probability II

Calculator Free

1. [5 marks: 3, 2]

   Given that $P(\overline{A \cup B}) = 0.2$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$;

   (a) find $P(B \mid A)$.

   (b) determine with reasons if the events A and B are independent.

---

2. [6 marks]

   Given that $P(A) = 0.4$, $P(C \mid A) = 0.3$, $P(C \mid \overline{A}) = 0.2$, find $P(A \mid C)$.

Calculator Free

3. [8 marks: 2, 2, 4]

It is known that P(A) = 0.6 and P(B) = 0.3. Find:

(a) $P(B \mid A)$ given that A and B are mutually exclusive.

(b) $P(A \mid B)$ given that A and B are independent.

(c) $P(B \mid A)$ given that $P(A \mid B) = 0.2$.

Calculator Free

4. [8 marks: 4, 2, 2]

Given that $P(A') = \frac{3}{10}$, $P(B|A) = \frac{2}{7}$ and $P(A|B) = \frac{1}{2}$:

(a) find P(B).

(b) find $P(A' \cap B')$.

(c) determine with reasons if A and B are independent.

Calculator Assumed

5. [5 marks: 1, 1, 3]

$P(A) = 0.3$, $P(B') = 0.4$ and $P(A \cap B) = k$:

(a) find in terms of k,

(i) $P(A \cap B')$.

(ii) $P(A' \cap B)$.

(b) find k given that $P(\overline{A \cup B}) = 0.18$.

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6. [4 marks]

Given that $P(A) = 0.5$, $P(B) = 0.8$ and $P(\overline{A} \cap \overline{B}) = 0.05$ and that the events A and B are independent, determine if these results are consistent with the rules of probability. Justify your answer.

Calculator Assumed

7. [10 marks: 1, 2, 4, 3]

Given that $P(A) = p + 0.2$ and $P(B) = p + 0.3$ and $P(A \cap B) = p$, calculate the value of p if:

(a) A and B are mutually exclusive events.

(b) $P(A \cup B) = 0.6$.

(c) A and B are independent events.

(d) $P(A \mid B) = 0.4$.

Calculator Assumed

8. [9 marks: 4, 3, 2]

A supply company checked its accounts and found that 8% of accounts were in arrears. 60% of the accounts were for sole traders while the other 40% were for companies. Only 2% of accounts were both in arrears and were accounts of sole traders.

(a) Find the probability of an account not being in arrears belonging to a sole trader.

(b) Of accounts in arrears, find the probability of the account belonging to a sole trader.

(c) Is the account status independent of the type of trader? Justify your answer.

Calculator Assumed

9. [7 marks: 1, 2, 2, 2] [TISC]

John drives to work each weekday morning. The route he takes passes through a set of traffic lights where he either has to stop at the lights or move through without stopping.

- If he has to stop at the lights, the probability that he will be late for work is 0.7.
- If he does not have to stop at the lights, the probability that he will be late for work is 0.2.

Overall, the probability that John will be late for work is 0.25.

(a) Find the probability that he will not be late for work if he did not have to stop at the traffic lights.

(b) Find the probability that John has to stop at the traffic lights.

(c) Determine with reasons if John being late is independent of whether he has to stop at the lights.

(d) Find the probability that John had to stop at the lights given that he was late for work.

Calculator Assumed

10. [10 marks: 2, 3, 3, 2] [TISC]

England and Germany play a series of two soccer matches. Each match does not end in a draw. The probability that Germany will win both matches is 0.42 and the probability that Germany will lose both matches is 0.135. The probability that Germany will win the first match is 0.7.

(a) Find the probability that England wins exactly one of the matches.

(b) Find the probability that England win the second match.

(c) Find the probability that Germany lose the first match given that they won the second match.

(d) Determine with reasons if the events that Germany win match one and the event that Germany win match two are independent.