

**ACADEMIC
EXCELLENCE
PROGRAM**

Methods Mock Test – Set B

Mathematics Methods Units 1 & 2

WACE Topic: Methods Year 11 – all topics

Medium and hard questions (challenging version). Questions marked ★ are hard.

Section One: Calculator-Free

58 marks — 60 minutes

Question 1

[8 marks: 3, 2, 3]

Points $A(1, 2)$ and $B(7, 10)$ are given.

- (a) Find the equation of the perpendicular bisector of AB .

- (b) Find the equation of the circle that has AB as a diameter.

- (c) Find the equation of the tangent to this circle at A , and the point where this tangent meets the y -axis.

Question 2 ★

[8 marks: 4, 4]

(a) Use first principles to show that the derivative of $f(x) = \frac{1}{\sqrt{x}}$ is $f'(x) = -\frac{1}{2x\sqrt{x}}$.

(b) Show that the tangent to the curve $y = x^3 - 3x$ at the point where $x = a$ meets the curve again where $x = -2a$.

Question 3 ★

[9 marks: 3, 3, 3]

(a) Prove that $\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = \frac{2}{\sin x}$.

(b) Solve $2\cos^2 x + 3\sin x = 3$ for $0 \leq x \leq 2\pi$.

(c) Given that $\sin x + \cos x = \frac{1}{2}$, find the exact value of $\sin x \cos x$ and hence of $\sin^3 x + \cos^3 x$.

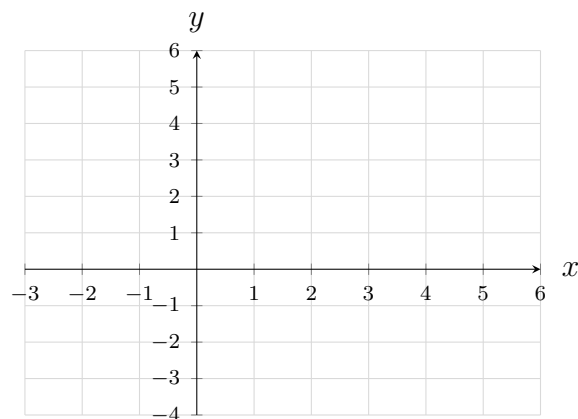
Question 4

[10 marks: 3, 4, 3]

(a) Solve $9^x - 4 \cdot 3^x + 3 = 0$.

(b) Given that $2^x + 2^{-x} = 3$, find the exact values of $4^x + 4^{-x}$ and $8^x + 8^{-x}$.

(c) Sketch $y = 4 - 2^{x-1}$ on the axes below, showing the asymptote and all intercepts.

**Question 5** ★

[7 marks: 4, 3]

(a) Find the coefficient of x^4 in the expansion of $(1 + 2x)^3(1 - x)^5$.

(b) Five boys and three girls stand in a row. In how many ways can they be arranged so that no two girls are next to each other?

Question 6 ★

[8 marks: 3, 3, 2]

- (a) Events A and B are independent, with $P(A) = 0.6$ and $P(A \cup B) = 0.8$. Find $P(B)$.
- (b) A bag contains 3 red balls and n blue balls. Two balls are drawn at random without replacement. The probability that both are red is $\frac{1}{15}$.
- (i) Show that $n^2 + 5n - 84 = 0$ and hence find n .
- (ii) Find the probability that exactly one ball is red.

Question 7 ★

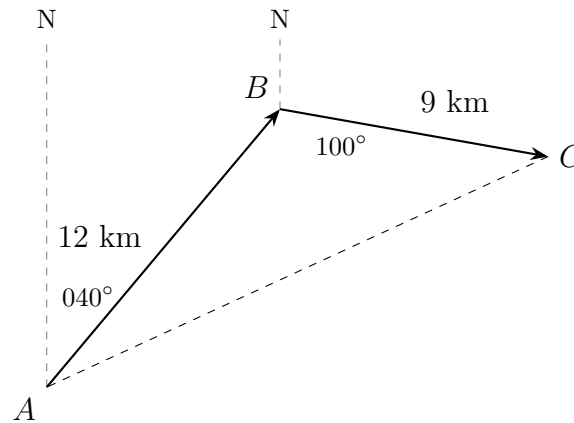
[8 marks: 2, 2, 4]

- (a) Let $f(x) = \frac{2x+1}{x-2}$. State the domain and range of f .
- (b) Find $f^{-1}(x)$ and hence show that $f(f(x)) = x$ for all x in the domain.
- (c) Let $p(x) = \sqrt{x-1}$ and $q(x) = x^2 + 2$. Find $p(q(x))$ and $q(p(x))$ (with its domain). Hence solve $p(q(x)) = q(p(x))$, explaining your answer.

End of Section One

Section Two: Calculator-Assumed **95 marks — 100 minutes****Question 8***[8 marks: 1, 2, 3, 2]*

A boat sails 12 km from A to B on a bearing of 040° , then 9 km from B to C on a bearing of 100° .



- (a) Find $\angle ABC$.
- (b) Find the distance AC , to the nearest 0.1 km.
- (c) Find the bearing of C from A and the bearing of A from C , to the nearest 0.1° .
- (d) Find the shortest distance from B to the line AC , to the nearest 0.1 km.

Question 9 ★

[8 marks: 2, 4, 2]

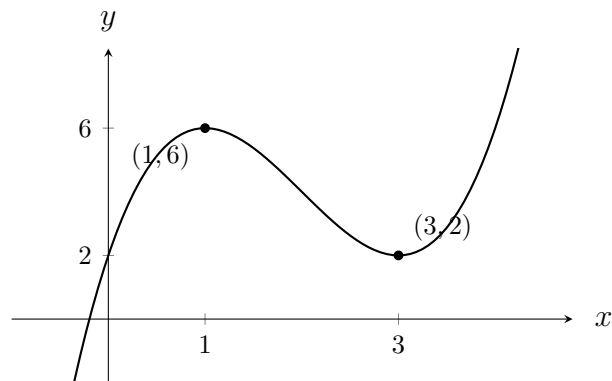
A closed cylindrical can has volume 500 cm^3 , radius $r \text{ cm}$ and height $h \text{ cm}$. The curved surface costs 0.4 cents per cm^2 to make, and the circular top and base cost 1 cent per cm^2 .

- (a) Show that the total cost is $C = \frac{400}{r} + 2\pi r^2$ cents.
- (b) Use calculus to find the value of r that minimises C , justify that it is a minimum, and find the minimum cost to the nearest cent.
- (c) Show that for this minimum cost, $h = 5r$.

Question 10 ★

[10 marks: 5, 2, 3]

A cubic $f(x) = ax^3 + bx^2 + cx + d$ has turning points at $(1, 6)$ and $(3, 2)$, as shown.

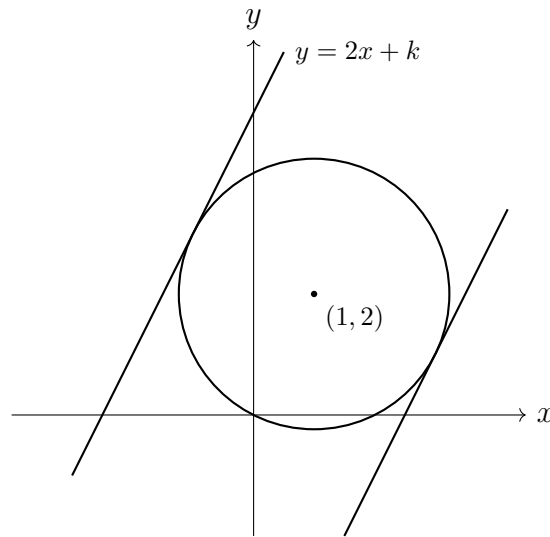


- (a) Find the values of a , b , c and d .
- (b) For what values of k does $f(x) = k$ have exactly one solution, two solutions, three solutions?
- (c) A line through the origin is tangent to the curve at the point where $x = t$, with $t > 0$. Show that $t^3 - 3t^2 - 1 = 0$ and hence find the equation of this tangent, with coefficients to 3 decimal places.

Question 11 ★

[9 marks: 4, 3, 2]

The lines $y = 2x + k$ are tangent to the circle $(x - 1)^2 + (y - 2)^2 = 5$ for two values of k .



- (a) Show that $5x^2 + (4k - 10)x + (k - 2)^2 - 4 = 0$ and hence find the two values of k .
- (b) Find the coordinates of the two points of tangency, and show that the line joining them passes through the centre of the circle.
- (c) When $k = 0$ the line cuts the circle at two points. Find the length of the chord between them.

Question 12 ★

[9 marks: 3, 3, 3]

(a) Solve $3 \sin 2x = 2 \cos x$ for $0 \leq x \leq 2\pi$, to 3 decimal places where necessary.

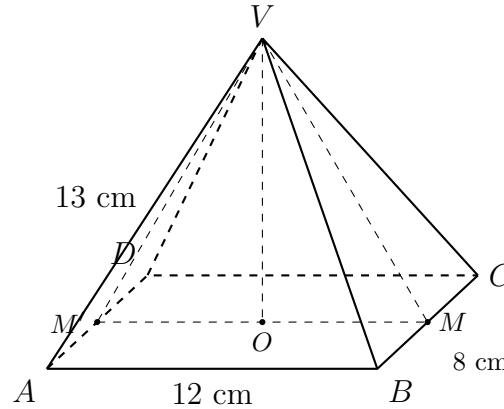
(b) Use $\sin(A + B)$ and $\cos(A + B)$ to show that $\sin 3x = 3 \sin x - 4 \sin^3 x$.

(c) Hence solve $\sin 3x = 2 \sin x$ for $0 \leq x \leq \pi$.

Question 13 ★

[9 marks: 2, 2, 2, 3]

$VABCD$ is a pyramid with a rectangular base $ABCD$, where $AB = 12$ cm and $BC = 8$ cm. The apex V is directly above the centre O of the base and each sloping edge is 13 cm. M and M' are the midpoints of BC and AD respectively.



- Find the height VO , to the nearest 0.1 cm.
- Find the angle between the edge VA and the base.
- Find $\angle AVB$.
- Find the angle between the faces VBC and VAD , i.e. $\angle MVM'$.

Question 14*[9 marks: 2, 1, 2, 2, 2]*

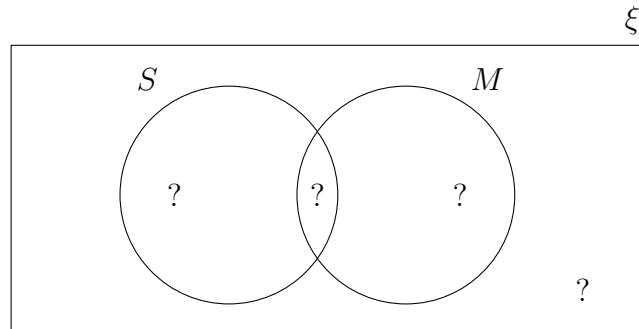
The number of bacteria in a culture t hours after it is started is $N(t) = 200 + 60t^2 - 2t^3$, for $0 \leq t \leq 20$.

- (a) Find the average rate of change of N over the first 10 hours.
- (b) Find the instantaneous rate of change of N at $t = 4$.
- (c) Find when the rate of growth is greatest and state its value.
- (d) Find the maximum population and when it occurs.
- (e) Find the times at which the instantaneous rate of change equals the average rate found in part (a).

Question 15

[10 marks: 2, 1, 2, 2, 3]

A survey of 120 students found that 70 play sport (S), 50 play music (M) and 20 do both.



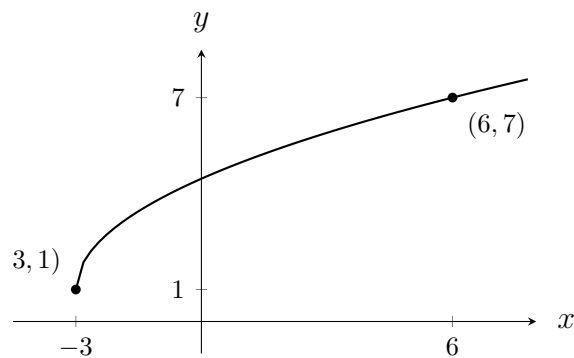
- (a) Complete the Venn diagram.
- (b) Find $P(M \mid S)$.
- (c) Are S and M independent? Justify with calculations.
- (d) Two students are chosen at random without replacement. Find the probability that neither plays sport nor music.
- (e) Given that at least one of two randomly chosen students plays sport, find the probability that both do.

Question 16

[9 marks: 4, 2, 3]

- (a) The mass of a radioactive sample is $M(t) = M_0 a^t$ grams after t years. After 1 year the mass is 60 g and after 3 years it is 38.4 g. Find M_0 and a , and the time for the mass to fall to 20 g.

- (b) The graph of $y = p\sqrt{x+q} + r$ is shown. Find p , q and r .

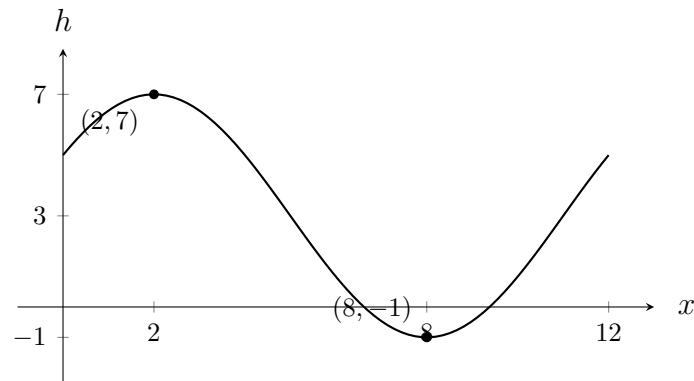


- (c) Find the point(s) of intersection of this curve and the line $y = x - 2$, explaining any rejected solution.

Question 17 ★

[8 marks: 3, 2, 3]

The graph of $h(x)$ for $0 \leq x \leq 12$ has a maximum at $(2, 7)$ and a minimum at $(8, -1)$.



- (a) Find the equation of h in the form $h(x) = a \cos(b(x - c)) + d$ with $a, b > 0$.
- (b) Solve $h(x) = 5$ for $0 \leq x \leq 12$.
- (c) A second function is $g(x) = 2 \sin\left(\frac{\pi x}{6}\right) + 3$. Find, to 2 decimal places, the values of x in $0 \leq x \leq 12$ where the graphs of g and h intersect.

Question 18

[6 marks: 3, 3]

- (a) Find the values of k for which $x^2 + kx + k + 3 = 0$ has no real roots.
- (b) The line $y = mx - 4$ is tangent to the parabola $y = x^2 + 3x$. Find the possible values of m .

End of Questions