

**ACADEMIC
EXCELLENCE
PROGRAM**

Mock Test – Set A

Vectors, Trigonometry & Complex Numbers

WACE Topic: Specialist Year 11 – Units 1 & 2

Questions marked ★ are harder / extension.

Section One: Calculator-Free

49 marks — 50 minutes

Question 1

[7 marks: 2, 2, 3]

Let $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j}$.

- (a) Determine $|\mathbf{a}|$ and hence the unit vector in the direction of $\mathbf{a}$.

- (b) Find a vector of magnitude 15 that points in the *opposite* direction to $\mathbf{a}$.

- (c) Determine the value of k so that $\mathbf{a} + k\mathbf{b}$ is parallel to $\mathbf{i} + \mathbf{j}$.

Question 2 ★

[8 marks: 2, 4, 2]

In triangle OAB , $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Point M is the midpoint of OB , and point P lies on AB so that $AP : PB = 2 : 1$. Lines AM and OP intersect at X .

- (a) Express $\overrightarrow{AM}$ and $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

- (b) Given $\overrightarrow{AX} = \lambda \overrightarrow{AM}$ and $\overrightarrow{OX} = \mu \overrightarrow{OP}$, where λ, μ are constants, find the values of λ and μ .

- (c) Hence state the ratios $AX : XM$ and $OX : XP$.

Question 3*[7 marks: 2, 3, 2]*

- (a) Prove the identity $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$.

- (b) Solve $2 \sin^2 x + \cos x - 1 = 0$ for $0 \leq x \leq 2\pi$.

- (c) Use a compound angle formula to find the exact value of $\sin 75^\circ$.

Question 4 ★*[7 marks: 3, 2, 2]*

- (a) Express $\sqrt{3} \sin x - \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

- (b) Hence solve $\sqrt{3} \sin x - \cos x = 1$ for $0 \leq x \leq 2\pi$.

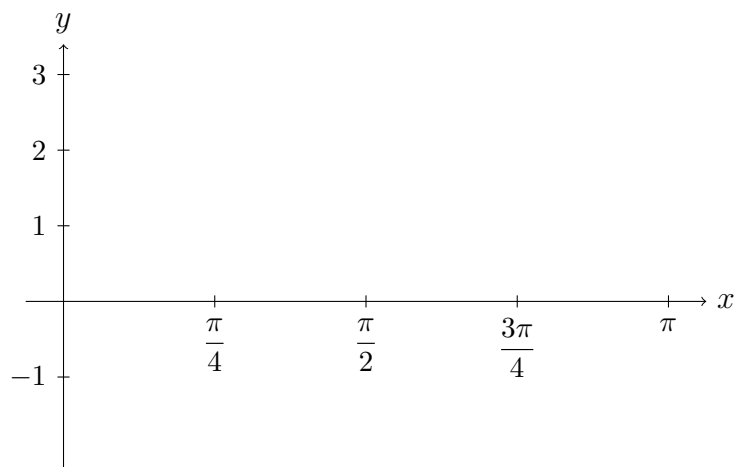
- (c) State the maximum value of $\sqrt{3} \sin x - \cos x$ and the smallest positive value of x at which it occurs.

Question 5

[6 marks: 2, 4]

Consider the function $f(x) = 1 - 2 \cos 2x$ for $0 \leq x \leq \pi$.

- (a) State the period, the amplitude and the range of f .
- (b) Sketch the graph of $y = f(x)$, labelling all intercepts with the axes and the coordinates of all turning points.

**Question 6**

[7 marks: 2, 2, 3]

- (a) Express $\frac{2 + 3i}{1 - i}$ in the form $a + bi$.

- (b) Solve $z^2 - 4z + 13 = 0$ for $z \in \mathbb{C}$.

- (c) Show that $z = 1$ is a root of $z^3 - 5z^2 + 17z - 13 = 0$ and hence find all three roots.

Question 7 ★

[7 marks: 4, 3]

- (a) Determine the complex number z that satisfies $3z - i\bar{z} = 11 - 9i$.

- (b) On an Argand diagram, show the points $z_1 = 3 + i$, $z_2 = -1 + 2i$, $z_1 + z_2$ and $z_1 - z_2$. Name the shape formed by 0 , z_1 , $z_1 + z_2$ and z_2 , and find the exact value of $|z_1 - z_2|$.

End of Section One

Section Two: Calculator-Assumed**60 marks — 70 minutes****Question 8***[7 marks: 3, 2, 2]*

A boat can travel at 6 m s^{-1} in still water. It is to cross a river, 120 m wide, whose banks run east–west. The river flows due east at 2.5 m s^{-1} . The boat wants to reach the point on the north bank directly opposite its starting point.

- (a) On what bearing (to the nearest 0.1°) should the boat be steered?

- (b) How long, to the nearest second, does the crossing take?

- (c) If instead the boat is steered due north, how far downstream does it land?

Question 9*[9 marks: 3, 2, 3, 1]*

Points $A(1, 2)$, $B(7, 5)$ and $C(4, 9)$ are given.

- (a) Find $\overrightarrow{AB}$, $\overrightarrow{AC}$ and the size of $\angle BAC$ to the nearest 0.1° .

- (b) Determine the vector projection of $\overrightarrow{AC}$ onto $\overrightarrow{AB}$.

- (c) Hence find the shortest distance from C to line AB and the area of triangle ABC .

- (d) Find the coordinates of D such that $ABCD$ is a parallelogram.

Question 10 ★

[9 marks: 4, 2, 3]

Parallelogram $OABC$ has $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$.

- (a) Use the scalar product to prove that the diagonals of $OABC$ are perpendicular if and only if $|\mathbf{a}| = |\mathbf{b}|$.

(b) Show that $|\overrightarrow{OB}|^2 + |\overrightarrow{AC}|^2 = 2(|\mathbf{a}|^2 + |\mathbf{b}|^2)$.

- (c) The sides of a parallelogram are 5 cm and 7 cm with an angle of 60° between them. Find the lengths of both diagonals to 2 decimal places.

Question 11

[6 marks: 4, 2]

Two forces act on a particle: F_1 of magnitude 40 N on bearing 030° , and F_2 of magnitude 55 N on bearing 140° .

- (a) Find the magnitude and bearing of the resultant force $\mathbf{R} = F_1 + F_2$.
- (b) A third force F_3 is added so that the particle is in equilibrium. State its magnitude and bearing.

Question 12

[5 marks]

Solve $\tan 2x = 3 \tan x$ for $0 \leq x \leq \pi$, giving exact values.

Question 13 ★

[6 marks: 3, 3]

(a) Prove that $\sin 3x = 3 \sin x - 4 \sin^3 x$.

(b) Hence solve $\sin 3x = 2 \sin x$ for $0 \leq x \leq 2\pi$.

Question 14

[6 marks: 3, 3]

The depth of water h metres in a harbour t hours after midnight is modelled by $h(t) = a \cos(b(t - c)) + d$. High tide of 4.8 m occurs at 02:00, the following low tide of 1.2 m occurs 6.2 hours later.

(a) Find the values of a , b , c and d (give b to 4 decimal places).

(b) A boat needs a depth of at least 2.4 m to move safely. For how many hours in one tidal cycle is it safe to move? Give your answer to the nearest minute.

Question 15

[6 marks: 4, 2]

The cubic $P(z) = z^3 + az^2 + bz + 10$ has real coefficients a, b , and $1 + 2i$ is one of its roots.

(a) Find a , b and all three roots of $P(z)$.

(b) Hence factorise $P(z)$ into a linear factor and a quadratic factor with real coefficients.

Question 16

[6 marks: 2, 1, 3]

Let $w = \frac{(1 + 2i)^2}{2 - i}$.

(a) Write w in the form $a + bi$.

(b) Find $|w|$ exactly.

(c) Given that w is a root of $z^2 + pz + q = 0$ where $p, q \in \mathbb{R}$, find p and q and state the other root.

End of Questions