

**ACADEMIC
EXCELLENCE
PROGRAM**

Mock Test – Set B

Vectors, Trigonometry & Complex Numbers

WACE Topic: Specialist Year 11 – Units 1 & 2

Questions marked ★ are harder / extension.

Section One: Calculator-Free

49 marks — 50 minutes

Question 1

[7 marks: 2, 2, 3]

Points $A(-1, 2)$, $B(3, -1)$ and $C(6, 3)$ are given.

(a) Find $\overrightarrow{BA}$ and $\overrightarrow{BC}$ and show that $|\overrightarrow{BA}| = |\overrightarrow{BC}|$.

(b) Use the scalar product to show that $\angle ABC = 90^\circ$.

(c) Find the coordinates of D so that $ABCD$ is a square, and state the area of the square.

Question 2 ★

[8 marks: 4, 4]

(a) AB is a diameter of a circle with centre O . Let $\overrightarrow{OA} = \mathbf{a}$, so $\overrightarrow{OB} = -\mathbf{a}$, and let P be any other point on the circle with $\overrightarrow{OP} = \mathbf{p}$. Use the scalar product to prove that $\angle APB = 90^\circ$.

- (b) The quadrilateral $PQRS$ has position vectors $\mathbf{p}, \mathbf{q}, \mathbf{r}, \mathbf{s}$. The midpoints of PQ , QR , RS and SP are K, L, M, N respectively. Prove that $KLMN$ is a parallelogram.

Question 3

[7 marks: 2, 2, 1, 2]

Angle x is acute with $\sin x = \frac{3}{5}$, and angle y is obtuse with $\cos y = -\frac{5}{13}$.

- (a) Find $\sin(x + y)$.
- (b) Find $\cos(x + y)$.
- (c) Hence write down $\tan(x + y)$.
- (d) Is $x + y$ acute, obtuse or reflex? Justify your answer using your results.

Question 4

[7 marks: 2, 3, 2]

- (a) Express $\sin 5x + \sin x$ as a product.
- (b) Hence solve $\sin 5x + \sin x = 0$ for $0 \leq x \leq \pi$.
- (c) Use a product-to-sum identity to evaluate $\cos 75^\circ \cos 15^\circ$ exactly.

Question 5 ★

[7 marks: 3, 3, 1]

(a) Prove that $\cot \theta - \tan \theta = 2 \cot 2\theta$.

(b) Hence solve $\cot \theta - \tan \theta = 2\sqrt{3}$ for $0 < \theta < \pi$, $\theta \neq \frac{\pi}{2}$.

(c) Write down the exact value of $\cot \frac{\pi}{8} - \tan \frac{\pi}{8}$.

Question 6

[7 marks: 1, 3, 2, 1]

(a) Expand $(3 - 2i)^2$.

(b) Hence solve $z^2 + z - 1 + 3i = 0$ for $z \in \mathbb{C}$. (Hint: compute the discriminant.)

(c) Explain why the two roots found in part (b) are not complex conjugates of each other.

(d) Find the exact distance between the two roots on the Argand diagram.

Question 7 ★

[7 marks: 3, 4]

(a) Find all complex numbers $z = a + bi$ with $a, b \in \mathbb{R}$ such that $z^2 = 3 + 4i$.

(b) Hence solve $2z^2 + 2z - 1 - 2i = 0$.

End of Section One

Section Two: Calculator-Assumed**67 marks — 75 minutes****Question 8***[8 marks: 3, 4, 1]*

Ship P is at the origin and sails at 16 km h^{-1} on a bearing of 030° . Ship Q is initially 30 km due east of P and sails at 12 km h^{-1} on a bearing of 300° .

- (a) Find the velocity of Q relative to P : give its magnitude and bearing (to 0.1°).

- (b) Determine when the ships are closest, to the nearest minute, and the minimum distance between them to 1 decimal place.

- (c) Will the ships collide? Justify.

Question 9*[9 marks: 2, 2, 3, 2]*

Let $\mathbf{a} = \begin{pmatrix} 2 \\ m \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} m-3 \\ 4 \end{pmatrix}$, where m is a constant.

- (a) Find m if $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

- (b) Find the exact values of m for which $\mathbf{a}$ and $\mathbf{b}$ are parallel.

- (c) When $m = 2$, find the angle between $\mathbf{a}$ and $\mathbf{b}$ to the nearest 0.1° .

- (d) When $m = 2$, resolve $\mathbf{a}$ into components parallel and perpendicular to $\mathbf{b}$.

Question 10 ★

[7 marks: 4, 3]

An aircraft has an airspeed of 250 km h^{-1} . A wind of 50 km h^{-1} blows *from* a bearing of 040° . The pilot wishes to fly 600 km along a track on bearing 120° .

(a) Find the bearing on which the aircraft must be headed, to the nearest 0.1° .

(b) Find the time taken for the flight, giving your answer in hours and minutes.

Question 11 ★

[7 marks: 2, 2, 3]

In triangle OAB , $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $|\mathbf{a}| = 4$, $|\mathbf{b}| = 6$ and $\angle AOB = 60^\circ$. Point P is the midpoint of AB .

(a) Find $|\overrightarrow{AB}|$ exactly.

(b) Find $|\overrightarrow{OP}|$ to 2 decimal places.

(c) Find the angle between $\overrightarrow{OP}$ and $\overrightarrow{AB}$ to the nearest 0.1° .

Question 12

[8 marks: 4, 4]

(a) Solve $3 \cos 2x + 5 \cos x + 2 = 0$ for $0 \leq x \leq 2\pi$, giving answers to 3 decimal places where not exact.

- (b) Solve $\sin x + \sin 3x = \cos x$ for $0 \leq x \leq 2\pi$, giving exact values.

Question 13*[8 marks: 2, 3, 3]*

A Ferris wheel has radius 25 m and its centre is 30 m above the ground. It completes one revolution every 4 minutes. A rider boards at the lowest point at $t = 0$ minutes.

- (a) Write an equation for the rider's height h metres above the ground at time t minutes.
- (b) Sketch $h(t)$ for $0 \leq t \leq 8$, labelling the maximum and minimum heights and the times at which they occur.
- (c) During one revolution, for how long (in minutes and seconds) is the rider at least 40 m above the ground?

Question 14 ★*[8 marks: 3, 3, 2]*

- (a) Express $5 \cos x + 12 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give α in radians to 4 decimal places.
- (b) Solve $5 \cos x + 12 \sin x = 6.5$ for $0 \leq x \leq 2\pi$.

- (c) The function $f(x) = \frac{26}{5 \cos x + 12 \sin x + 15}$ is defined for all real x . Find its maximum and minimum values.

Question 15*[6 marks: 3, 3]*

The cubic $P(z) = z^3 + az^2 + bz - 13$ has real coefficients and $z = 3 + 2i$ is one of its roots.

- (a) Find the values of a and b , and the other two roots.
- (b) Show the three roots on an Argand diagram and find the area of the triangle whose vertices are the three roots.

Question 16 ★*[6 marks: 3, 2, 1]*

- (a) Solve $z + \frac{1}{z} = 1$ for $z \in \mathbb{C}$.

- (b) Show that each solution in part (a) satisfies $z^3 = -1$.

- (c) Hence write down the value of z^6 for these solutions.

End of Questions