

01 Combinatorics I

Calculator Free

1. [9 marks: 2, 2, 2, 3]

Find k if:

(a) $\frac{10!}{k!} = 720$

(b) $\frac{k!}{8!} = 990$

(c) ${}^kP_3 = 210$

(d) $\frac{10! - k!}{k!} = 30\,239$

Calculator Free

2. [4 marks: 1, 1, 1, 1]

The letters of the word RICHES are rearranged in a line. No letter is used more than once. Write mathematical expressions for:

- (a) the total number of possible arrangements.
 - (b) the number of arrangements with the letters I and E are adjacent.
 - (c) the number of arrangements with the letters I and E not adjacent
 - (d) the number of arrangements where the vowels are adjacent and the letters C and H are adjacent.
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3. [4 marks: 1, 1, 1, 1]

Codes are formed by using the letters of the word MATHS, with no letter being used more than once:

- (a) How many five letter codes start with M?
- (b) How many five letter codes have the letters A and M together?
- (c) How many four letter codes start with the letter M ?
- (d) How many three letter codes end with the letter S.

Calculator Assumed

4. [4 marks: 1, 1, 1, 1]

Three year 11 students and four year 12 students are to be arranged in a line.

- (a) In how many ways can this be done?
- (b) In how many arrangements are all the year 11 students next to each other?
- (c) How many arrangements have all students of the same year group adjacent to each other?
- (d) How many arrangements have no student of the same year group adjacent to each other?

5. [9 marks: 1, 2, 3, 3]

Six digit numbers are formed using the digits 1, 2, 3, 4, 5, and 6. Each digit is used only once.

- (a) How many six digit numbers are possible?
- (b) How many six digit even numbers are possible?
- (c) How many five digit even numbers greater than 40 000 are possible?
- (d) How many even numbers greater than 40 000 are possible?

Calculator Assumed

6. [7 marks: 2, 2, 3]

Passwords consisting of between 8 and 12 characters inclusive are to be created using the letters of the alphabet (case sensitive) and the digits 0 to 9 inclusive.

(a) Write mathematical expressions for the number of possible passwords if:

(i) no character can be used more than once.

(ii) repetition of characters are permitted.

(b) A computer program is capable of checking 4 billion (1×10^9) passwords per second. How long will the computer take to check all the possible passwords in part (a) (ii)? Give your answer in years.

7. [4 marks: 2, 2]

The length of a password is determined by the number of characters in the password. Using the digits 0 to 9 inclusive and the case sensitive letters of the alphabet, find the minimum length of a password if the number of possible passwords is to exceed 1 trillion (1×10^{12}) if:

(a) repetition of characters is not permitted.

(b) repetition of characters is permitted.

02 Combinatorics II

Calculator Assumed

1. [5 marks: 1, 2, 2]

A random survey involving 200 students revealed the following. 38 students did not have any calculator (scientific or CAS) with them. 142 students had a CAS calculator with them. 52 students had a scientific calculator with them. How many students had:

- (a) at least one calculator with them?
- (b) both types of calculators with them?
- (c) exactly one type of calculator with them?

2. [5 marks: 3, 2]

80 students were asked in a survey if they had previously broken a leg and/or broken an arm. Information from the survey indicated that 40% of students had suffered broken arms, one quarter of those surveyed had suffered broken legs and 40% had not previously had a broken arm or leg. How many students had:

- (a) broken an arm and a leg?
- (b) broken an arm but not a leg?

Calculator Assumed

3. [4 marks: 1, 1, 2]

In a group of 40 students, there were 10 boys who were colour vision deficient (CVD) and 15 girls who were not CVD. There were as many boys who were not CVD as there were boys who were CVD. How many of these students:

- (a) were boys?
 - (b) were either boys or colour vision deficient?
 - (c) were colour vision deficient?
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4. [5 marks: 2, 2, 1]

In a survey of teachers teaching mathematics to year nines, 15 teachers had a mathematics degree, 55 men teachers did not have a mathematics degree and 10 women teachers had a mathematics degree. There were as many teachers who were either female or had a mathematics degree as there were men teachers.

- (a) How many male teachers were there?
 - (b) How many female teachers were there?
 - (c) How many teachers were surveyed?
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Calculator Assumed

5. [7 marks: 1, 1, 1, 1, 2, 1]

Flags of 10 different nations including that of Australia and New Zealand are to be flown from 10 flag poles set in a line. The flag poles are labelled poles 1 to 10. How many ways are there of assigning a flag to each of these poles if the:

- (a) Australian flag must be flown from pole 1?
- (b) Australian flag or New Zealand flag must be flown from pole 1?
- (c) Australian flag and the New Zealand flag must be flown from poles 1 and 10 respectively?
- (d) Australian flag must be flown from pole 1 and the New Zealand flag must not to be flown from pole 10.
- (e) Australian flag must be flown from pole 1 or the New Zealand flag must be flown from pole 10.
- (f) Australian flag must be flown from pole 1 or the New Zealand flag must be flown from pole 10 but not both at the same time.

Calculator Assumed

6. [6 marks: 2, 2, 2]

Twelve different coloured light bulbs, including a red bulb and a blue bulb are to be fitted into bulb sockets installed along a straight edge of a patio running East to West. The bulb socket at the extreme Eastern end is labelled E and the bulb socket at the extreme Western end is labelled W. Determine the number of arrangements with:

- (a) the red light bulb not fitted into bulb sockets E or W.
- (b) the red light bulb and the blue light bulb not fitted into bulb sockets E or W.
- (c) the red light bulb or the blue light bulb not fitted into bulb sockets E or W.

7. [7 marks: 2, 2, 3]

Ten potted plants including four pots of roses of different shades of red and three pots of azaleas (each of a different colour) are to be arranged in a line along a footpath. How many arrangements will have:

- (a) the potted roses adjacent to each other?
- (b) the potted azaleas adjacent to each other?
- (c) the roses adjacent to each other or the azaleas adjacent to each other?

Calculator Assumed

8. [6 marks: 2, 2, 2]

An analysis was conducted on the tickets bought by 150 patrons of the Perth International Festival. The survey was restricted to the three categories: Classical Music, Contemporary Music, Theatre, Circus & Dance (TCD).

- All had bought tickets to at least one of these categories.
- 30 had tickets to all categories.
- A total of 100 had tickets to a Classical Music event.
- A total of 90 had tickets to a Contemporary Music event.
- 80 had tickets to a Classical Music event as well as a TCD event.
- 50 had tickets to a Classical Music as well as a Contemporary Music event.
- 30 had tickets to a Contemporary Music as well as a TCD event.

(a) How many patrons had tickets to a Classical Music and a Contemporary Music event but not a Theatre, Circus & Dance event?

(b) How many patrons had tickets to only a Classical Music event?

(c) How many patrons had tickets to a Theatre, Circus & Dance event?

Calculator Assumed

9. [6 marks: 3, 3]

A survey was conducted on 100 travellers at an airport.

- 62 had been vaccinated against yellow fever (Y).
- 80 had been vaccinated against typhoid (T).
- 82 had been vaccinated against malaria (M).
- 8 had not been vaccinated against any of these three diseases.
- All who had vaccinations had been vaccinated against more than one of these three diseases.

(a) How many travellers were vaccinated against yellow fever and typhoid but not malaria?

(b) How many travellers were vaccinated against yellow fever, typhoid and malaria?

Calculator Assumed

10. [6 marks: 1, 4, 1]

50 students were asked if they had attended the open-day sessions at the University of Western Australia (W), Murdoch University (M) and Curtin University (C).

- 5 students did not attend any of these sessions.
- 20 students attended the open day session at W.
- 23 students attended the open day session at M.
- 30 students attended the open day session at C.
- 3 students attended the open day sessions at W and M but not at C.
- 7 students attended the open day sessions at C and M but not at W.
- 12 students attended the open day sessions at C and W but not at M.

(a) How many students attended sessions at exactly 2 of these universities?

(b) How many students attended sessions at no more than 2 of these universities?

(c) How many students attended sessions at exactly one of these 3 universities?

Calculator Assumed

11. [8 marks: 1, 1, 1, 2, 1, 2]

Seven students including Amy, Brian and Catherine are to be arranged in a line. How many possible arrangements are there with:

- (a) Amy or Brian or Catherine on the extreme left?
- (b) Amy on the extreme left?
- (c) Amy on the extreme left and Catherine on the extreme right?
- (d) Amy on the extreme left or Catherine on the extreme right?
- (e) Amy on the extreme left, Brian in the middle and Catherine on the extreme right?
- (f) Amy on the extreme left or Brian in the middle or Catherine on the extreme right?

Calculator Assumed

12. [11 marks: 1, 1, 2, 3, 2, 2]

Ten different coloured balls, including a red, a blue and a green ball, are to be placed in ten different boxes labelled A to J. One ball is to be placed in each box. How many ways are there of placing the balls (one in each box) with:

(a) the red ball in box A?

(b) the red ball in box A and the blue ball in box B?

(c) the red ball in box A or the blue ball in box B?

(d) the red ball in box A or the blue ball in box B or the green ball in box C?

(e) the red ball in box A but the blue ball not in box B?

(f) the green ball in box C but the red ball not in box A and the blue ball not in box B?

Calculator Assumed

13. [8 marks: 1, 1, 1, 1, 2, 2]

Consider the set of integers between 1 000 and 9 999 inclusive.
How many integers in this set:

(a) are divisible by 2?

(b) are divisible by 3?

(c) are divisible by 2 and 3?

(d) are divisible by 2 and 6?

(e) are divisible by 2 or 3?

(f) are divisible by 2 or 3 but not both?

Calculator Assumed

14. [7 marks: 1, 1, 1, 2, 2]

Consider the set of integers between 500 and 5 000 inclusive.
How many integers in this set:

(a) are divisible by 5?

(b) are divisible by 10?

(c) are divisible by 5 and 10?

(d) are divisible by 5 or 10?

(e) are divisible by 5 or 10 and is an even number?

Calculator Assumed

15. [6 marks: 3, 3]

Consider the set of integers between 1 000 and 5 000 inclusive.

- (a) Complete the following table listing the number of multiples of n within this set for the given values of n .

n	Number of multiples of n in this set.
2	
3	
5	
6	
10	
15	
30	

- (b) Find the number of integers in this set that are multiples of 2, 3 or 5.

Calculator Assumed

16. [6 marks: 3, 3]

Consider the set of integers between 500 and 5 000 inclusive.

(a) Complete the following table.

Multiples of	Number of multiples in this set.
3	
5	
7	
3 and 5	
3 and 7	
5 and 7	
3 and 5 and 7	

(b) Find the number of integers in this set that are multiples of 3, 5 or 7.

Calculator Assumed

17. [6 marks: 3, 3]

Consider the set of integers between 2500 and 10 000 inclusive.

(a) Complete the following table.

Multiples of	Number of multiples in this set.
2	
4	
5	
2 and 4	
2 and 5	
4 and 5	
2 and 4 and 5	

(b) Find the number of integers in this set that are multiples of 2, 4 or 5.

03 Combinatorics III

Calculator Free

1. [8 marks: 2, 2, 2, 2]

(a) Determine the $\binom{9}{6}$.

(b) Evaluate $\binom{9}{3} + \binom{9}{4}$.

(c) Simplify ${}^{10}C_5 \times {}^5P_5$. You are not required to evaluate your answer.

(d) Given that ${}^5P_{r+1} = {}^5P_r$, find r .

Calculator Free

2. [13 marks: 2, 2, 2, 4, 3]

(a) Find n and r if ${}^nC_r = \frac{n \times (n-1) \times 98}{3 \times 2 \times 1}$.

(b) Find n and r if ${}^nP_r = 20 \times 19 \times 18 \times 17$

(c) Find a and b if ${}^{30}C_a = {}^{3a}C_b$.

(d) Find all possible values of a and b if ${}^{12}C_a = {}^{12}C_{2a+b}$.

(e) Find a possible set of values for a and b if $10 \times {}^9P_4 = 6 \times {}^aP_b$.

Calculator Free

3. [3 marks]

(a) How many ways are there of choosing 2 students from a group of 100 students?

(b) How many ways are there of choosing 98 students from a group of 100 students?

4. [7 marks: 2, 2, 3]

A media folder has 10 video-clips. How many ways are there of choosing:

(a) seven of these clips?

(b) seven or eight of these clips?

(c) at least one of these clips?

Calculator Assumed

5. [12 marks: 1, 3, 2, 3, 3]

A committee of 9 people are to be selected from 10 Labor, 8 Liberal and 5 Green politicians. How many different ways can the committee be selected if:

(a) there are no restrictions

(b) all three political parties are equally represented

(c) there are no Green representatives

(d) the Liberal representatives are in the (absolute) majority

(e) a husband and wife pair, Alex and Alice, cannot be in the same committee.

Calculator Assumed

6. [10 marks: 1, 2, 3, 4]

[TISC]

Wei has a collection of 20 stickers in her pink box and 25 stickers in her blue box. All these stickers are different from each other.

- (a) In how many ways can Wei pick 3 stickers from her pink box?

- (b) In how many ways can Wei pick 2 stickers from her blue box and arrange them in a line?

- (c) In how many ways can Wei pick 3 stickers from her pink box and 2 stickers from her blue box and arrange them in a line if:
 - (i) there are no restrictions as to how the stickers are arranged?

 - (ii) all the stickers from the blue box must be together?

Calculator Free

7. [12 marks: 1, 2, 2, 2, 2, 3]

In how many ways can the letters of the word **COMBINE** be arranged in a straight line (no letter may be used more than once):

- (a) using all seven letters?
- (b) using all seven letters and starting with the letter **C**?
- (c) using all seven letters and ending in **BONE**?
- (d) using only 5 letters at a time?
- (e) using all seven letters with the vowels in the first three places (from the left)?
- (f) using only two vowels and two consonants?

Calculator Free

8. [12 marks: 1, 2, 2, 3, 4]

Consider the letters of the word CONQUEST. Write mathematical expressions for the number of ways the letters of this word can be *rearranged*:

- (a) if the first letter must be a consonant.

- (b) if the first three letters must be consonants.

- (c) if one of the first three letters must be a consonant.

- (d) if at least one of the first three letters must be a consonant.

- (e) if the vowels are to be sandwiched between two consonants (that is, a vowel must be preceded by a consonant and this vowel must also be followed by a consonant).

Calculator Free

9. [5 marks: 1, 1, 3]

[TISC]

Jane wishes to create a password using the digits 1, 2, 3, 4, 5, 6 and the letters of her name J, A, N and E.

(a) How many eight character passwords can be created, if no character is used more than once?

(b) How many of the passwords in (a) include the letters J, A, N and E?

(c) How many passwords in (a) have digits adding up to exactly ten.

10. [6 marks: 3, 3]

A number is divisible by four if the last two digits of the number is divisible by four. For example, 4564 is divisible by four because the last two digits "64" is divisible by four; but 4502 is not divisible by four because the last two digits "02" is not divisible by four. Using the digits 0 to 9 inclusive, write an expression for the number of five digit numbers divisible by four that can be formed:

(a) if digits may be repeated?

(b) if no digit is to be repeated?

Calculator Assumed

11. [8 marks: 1, 1, 2, 2, 2]

A contingent of six athletes is to be formed from 10 swimmers, 12 cyclists and 8 gymnasts. How many different contingents can be formed if each contingent must have:

(a) 2 gymnasts?

(b) 3 swimmers

(c) 2 gymnasts and 3 swimmers

(d) 2 gymnasts or 3 swimmers?

(e) 2 gymnasts or 3 swimmers but not both?

Calculator Assumed

12. [7 marks: 3, 2, 2]

Five books are to be selected and arranged on a library display shelf. These five books are to be selected from a collection of 10 adult novels, 5 non-fiction books and 8 illustrated children's books.

(a) Complete the table below.

Composition of books on display shelf	Number of different arrangements
2 adult novels	
3 illustrated children's books	
2 adult novels and 3 illustrated children's book	

(b) Determine the number of possible arrangements with either 2 adult novels or 3 illustrated children's books.

(c) Determine the number of possible arrangements with either 2 adult novels or 3 illustrated children's books but not both.

Calculator Assumed

13. [8 marks: 3, 2, 3]

A password consisting of four characters is to be chosen from 10 digits, 26 upper case letters, 26 lower case letters and a set of 32 symbols.

(a) Complete the table below.

Composition of password	Number of different passwords
2 lower case letters	
2 symbols	
2 lower case letters and 2 symbols	

(b) Determine the number of possible passwords with either two lower case letters or two symbols.

(c) Determine the number of possible passwords with either two lower case letters or two upper case letters.

Calculator Assumed

14. [9 marks: 3, 2, 4]

A password consisting of four characters is to be chosen from 26 upper case letters, 26 lower case letters, 10 digits and a set of 32 symbols.

(a) Complete the table below.

Composition of password	Number of different passwords
Exactly 2 lower case letters	
Exactly 1 symbol	
Exactly 2 lower case letters and 1 symbol	

(b) Determine the number of possible passwords with either two lower case letters or one symbol.

(c) Determine the number of possible passwords with either one symbol or one upper case letter.

Calculator Assumed

15. [7 marks: 4, 3]

A sample of ten students is to be selected from a group of four year 8, five year 9, six year 10, seven year 11 and eight year 12 students.

(a) Complete the table below.

Composition of sample	Number of different samples
Five year 12 students.	
Three year 11 students.	
Two year 10 students.	
Five year 12 and three year 11 students.	
Five year 12 and two year 10 students.	
Three year 11 and two year 10 students.	
Five year 12 and three year 11 and two year 10 students.	

(b) Determine the number of possible samples with five year 12 or three year 11 or two year 10 students.

04 Combinatorics IV

Calculator Free

1. [6 marks: 1, 2, 1, 2]

A container has 10 different sized pairs of nuts and bolts with the nuts removed from the respective bolts.

- (a) Five bolts are randomly removed from this container. What is the minimum number of nuts that need to be removed from this container to ensure one matching pair of nut and bolt?

- (b) Six bolts are randomly removed from this container. What is the minimum number of nuts that need to be removed from this container to ensure two matching pairs of nut and bolt?

- (c) What is the minimum number of items that need to be removed from this container to ensure one matching pair of nut and bolt?

- (d) Fourteen items are randomly removed from the container.
 - (i) What is the minimum number of matching pairs of nuts and bolts?

 - (ii) What is the maximum possible number of matching pairs?

Calculator Free

2. [7 marks: 1, 2, 2, 2]

Twelve dog owners and their dogs (one dog per owner) meet at a dog park.

(a) Six dog owners are randomly chosen. What is the minimum number of dogs that need to be chosen to ensure a matching owner-dog pair?

(b) Seven dogs are randomly chosen. What is the minimum number of owners that need to be chosen to ensure three matching owner-dog pairs?

(c) A total of 15 owners and dogs were randomly selected.

(i) What is the minimum number of owner-dog pairs in this selection?

(ii) What is the maximum possible number of owner-dog pairs in this selection?

(d) If the owners came as couples, that is one dog per couple, what is the minimum number of persons and dogs that need to be chosen to ensure:

(i) a matching owner-dog pair?

(ii) more than two matching owner-dog pairs?

Calculator Free

3. [7 marks: 1 each]

A container has 5 red marbles, 6 green marbles and 9 yellow marbles. What is the minimum number of marbles that need to be drawn from this container to ensure:

- (a) a marble of each colour?
- (b) two marbles of each colour?
- (c) two red marbles?
- (d) two yellow marbles?
- (e) two green marbles?
- (f) two marbles of the same colour?
- (g) three marbles of the same colour?

Calculator Free

4. [7 marks: 1 each]

Dennis has 3 blue pens, 4 red pens and 5 black pens in his pencil case. What is the minimum number of pens that need to be drawn from the pencil case to ensure that:

- (a) a red pen is drawn?
- (b) a pen of each colour is drawn?
- (c) a red and a blue pen is drawn?
- (d) two blue pens and two red pens are drawn?
- (e) two red pens and two black pens are drawn
- (f) two blue pens and two black pens are drawn?
- (g) two pens of the same colour are drawn?

Calculator Assumed

5. [4 marks: 2, 2]

There are 25 students in a class.

- (a) Explain clearly why there must be at least 3 students that are born in a same month.
- (b) Explain clearly why there must be at least one month which is a birth month shared by no more than 2 students.

Calculator Assumed

6. [6 marks: 1, 1, 2, 2]

A class has 30 students.

(a) How many students need to be chosen to ensure that there are:
(i) two students who are born on the same day of the week?

(ii) five students who are born on the same day of the week?

(b) There are at least x students who are born on the same day of the week.
Find x . Justify your answer.

(c) There must be at least one day of the week which is the birth day
of no more than y students. Find y . Justify your answer.

Calculator Assumed

7. [7 marks: 1, 1, 1, 2, 2]

There are 300 students in a primary school.

- (a) How many students need to be chosen to ensure that there are:
- (i) two students with family names that start with the same letter?

 - (ii) three students with family names that start with the same letter?

 - (iii) six students with family names that start with the same letter?
- (b) There are at least x students with family names that start with the same letter. Find x . Justify your answer.
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- (c) There must be at least one letter that is the first letter of the family names of no more than y students. Find y . Justify your answer.