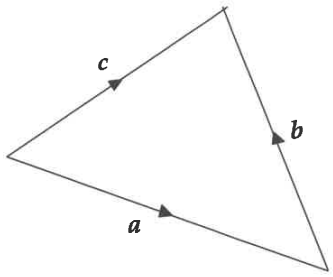
05 Addition & Subtraction of Vectors (Using Trigonometry)

Calculator Free

1. [2 marks: 1, 1]

Vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are as drawn in the accompanying diagram.

(a) Express $\boldsymbol{c}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.



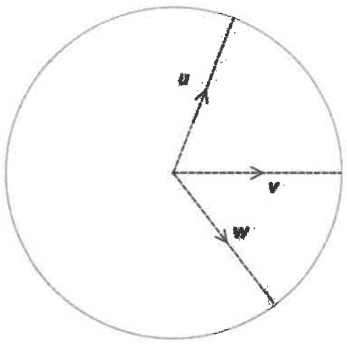
(b) Express $\boldsymbol{a}$ in terms of $\boldsymbol{b}$ and $\boldsymbol{c}$.

---

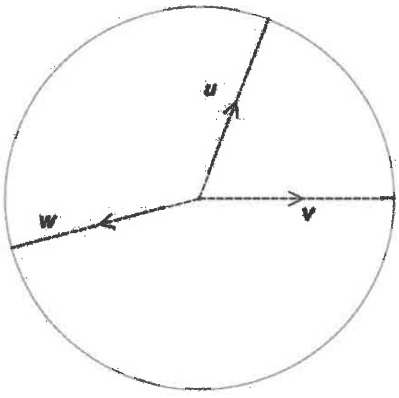
2. [4 marks: 2, 2]

The accompanying diagram shows the relative positions of the vectors $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$. $|\boldsymbol{u}| = |\boldsymbol{v}| = |\boldsymbol{w}|$.

(a) Sketch in the space provided below $\boldsymbol{u} + \boldsymbol{v} + \boldsymbol{w}$.



(b) Sketch in the space provided below $\boldsymbol{v} - \boldsymbol{u} - \boldsymbol{w}$.



Calculator Assumed

3. [7 marks: 3, 4]

The angle between $\boldsymbol{u}$ and $\boldsymbol{u} - \boldsymbol{v}$ is 30°, $|\boldsymbol{v}| = 5$ and $|\boldsymbol{u} - \boldsymbol{v}| = 10$.

(a) Draw a clearly labelled sketch of the vectors $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$.

(b) Use the rules of trigonometry to find $|\boldsymbol{u}|$ and the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$.

---

4. [6 marks: 2, 4]

An aircraft is flying with a speed of 400 kmh^{-1} along bearing 145°. The aircraft is buffeted by a strong wind of magnitude 80 kmh^{-1} blowing from bearing 240°.

(a) Draw a sketch to indicate the actual direction of the aircraft.

Calculator Assumed

4. (b) Find the ground speed and direction of the aircraft.

---

5. [6 marks: 3, 3]

A boy intends to swim across a river of width 20 metres to the opposite bank. The river flows at a steady rate of 1 kmh^{-1}. The boy can swim at a steady speed of 2 kmh^{-1}.

(a) In what direction should the boy be headed so that he ends up at the opposite bank directly opposite to where he started off ?

(b) Find the time taken for the swim in part (a).

Calculator Assumed

6. [5 marks]

A current is flowing in the direction N48°E at 10 kmh^{-1}. With what speed and in what direction should a naval vessel be travelling to achieve a resultant speed of 40 kmh^{-1} in the direction N30°W.

06 Components & Position Vectors I

Calculator Free

1. [8 marks: 2, 1, 1, 4]

   Given that $\boldsymbol{a} = -2\,\mathbf{i} + 6\,\mathbf{j}$ and $\boldsymbol{b} = 5\,\mathbf{i} - 4\,\mathbf{j}$, find:
   (a) $|\boldsymbol{a} + \boldsymbol{b}|$.

   (b) the unit vector parallel to $\boldsymbol{a} + \boldsymbol{b}$.

   (c) a vector that is parallel to $\boldsymbol{a} + \boldsymbol{b}$ but with a magnitude of 5.

   (d) $\boldsymbol{a}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$ where $\boldsymbol{p} = 2\mathbf{i} + \mathbf{j}$ and $\boldsymbol{q} = -3\mathbf{i} + 2\mathbf{j}$.

Calculator Free

2. [6 marks]

$\mathbf{OA} = 3\mathbf{i} + 10\mathbf{j}$, $\mathbf{OB} = 5\mathbf{i} + b\mathbf{j}$ and $\mathbf{OC} = 9\mathbf{i} + c\mathbf{j}$.
Find c in terms of b if A, B and C are collinear.

3. [7 marks]

Vector $a\mathbf{i} + (a + b)\mathbf{j}$ has a magnitude of 5 and is parallel to vector $4\mathbf{i} + 8\mathbf{j}$.
Find all possible values of a and b.

Calculator Free

4. [5 marks]

Vector $a\,\mathbf{i} + 10\mathbf{j}$ is of the same magnitude as $(b-10)\,\mathbf{i} + (a-2b)\,\mathbf{j}$ but acts in the opposite direction. Find the values of a and b.

5. [4 marks]

Vector $\begin{pmatrix} a \\ b \end{pmatrix}$ has magnitude 20 and is parallel to $\begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix}$.

Find the values of a and b.

6. [3 marks]

The point K divides the line segment AB internally in the ratio 4 : 1. Use a vector method to find the position vector of K if $\mathbf{OB} = -\mathbf{i} + 2\mathbf{j}$ and $\mathbf{AB} = 15\mathbf{i} - 5\mathbf{j}$.

Calculator Assumed

7. [3 marks]

The points P and Q have position vectors $5\mathbf{i} - 2\mathbf{j}$ and $-4\mathbf{i} + 5\mathbf{j}$ respectively. The point K is such that $\mathbf{PK} = -4\mathbf{QK}$. Find the position vector of K.

8. [4 marks]

It is known that $\mathbf{OA} = a\mathbf{i} + \mathbf{j}$ and $\mathbf{OB} = 4\mathbf{i} + b\mathbf{j}$.
K is a point such that AK: AB = 2 : 5 and $\mathbf{OK} = 4\mathbf{i} - 3\mathbf{j}$. Find a and b.

Calculator Assumed

9. [5 marks]

Vector $\boldsymbol{u}$ has magnitude 100 kmh^{-1} and acts in the direction 040°.
Vector $\boldsymbol{v}$ has magnitude 150 kmh^{-1} and acts in the direction 280°.
Let $\boldsymbol{i}$ be the unit vector in the West-East direction
and $\boldsymbol{j}$ be the unit vector in the South-North direction.

Use vector components to find the magnitude and direction of $\boldsymbol{u} - 2\boldsymbol{v}$.

10. [5 marks]

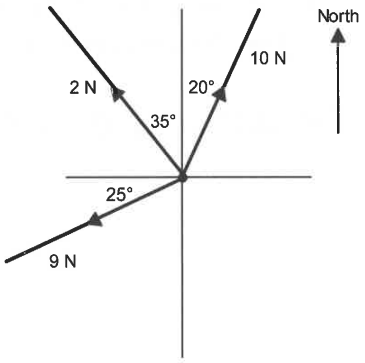
Given that $\boldsymbol{u} = \langle -4, 16 \rangle$ and $|\boldsymbol{v}| = 100$, find $\boldsymbol{v}$
if $\boldsymbol{u} + \boldsymbol{v}$ is to be in the same direction as the vector $\langle 2, 2 \rangle$.

07 Components & Position Vectors II

Calculator Assumed

1. [6 marks: 3, 3]

The diagram below shows the forces acting on a body. The forces are all on the same plane.



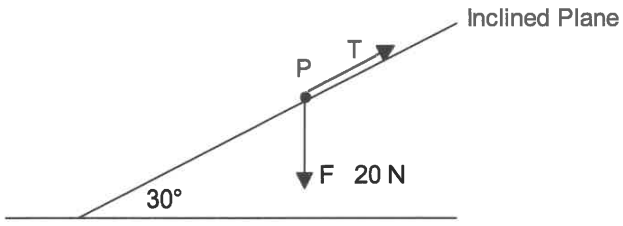
(a) Find the magnitude of the resultant.

(b) Find the magnitude and the direction of a single force that will keep this system in equilibrium.

Calculator Assumed

2. [5 marks: 2, 2, 1]

In the diagram below, a particle P is on a plane inclined at an angle of 30° to the horizontal. A vertical force F of magnitude 20 N is acting on P as shown. Force T parallel to the inclined plane is applied to prevent P from slipping down the inclined plane.



(a) Find the magnitude of the component of F parallel to the inclined plane.

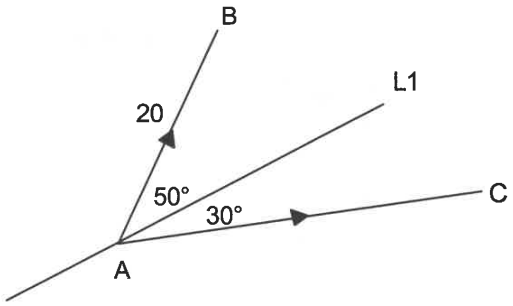
(b) Find the magnitude of the component of F perpendicular to the inclined plane.

(c) Find the magnitude of force T.

Calculator Assumed

3. [4 marks: 1, 1, 2]

In the diagram below, vector **AB** is of magnitude 20 units and is inclined at an angle of 50° to the line L1. Vector **AC** is inclined at angle of 30° to the line L1 as shown.



(a) Find the magnitude of the component of **AB** parallel to the line L1.

(b) Find the magnitude of the component of **AB** perpendicular to line L1.

(c) Find the magnitude of **AC** if the resultant of the vectors **AB** and **AC** is parallel to the line L1.

Calculator Assumed

4. [11 marks: 1, 1, 4, 2, 3]

A light plane can fly at 80 km per hour in still air. The pilot wishes to fly from O to a neighbouring airstrip Q, located 40 km from O in the direction 060°. A constant wind of 20 km per hour is blowing from the North. **i** and **j** are unit vectors in the Easterly and Northerly directions respectively.

(a) Write in terms of **i** and **j** the position vector of Q relative to O.

(b) Write in terms of **i** and **j** the velocity vector of the wind.

(c) Find the velocity vector the pilot should set so that the plane flies directly to Q.

(d) Find the resultant speed of the plane.

(e) Find the difference in flying time (to the nearest minute) caused by the wind.

Calculator Assumed

5. [8 marks: 4, 2, 2]

A helicopter capable of flying at a speed of 100 km per hour in still air, takes off from O for a mining town located at A. The position vector of A relative to O is $200\mathbf{i} - 300\mathbf{j}$ km. Throughout the journey, the helicopter encounters a wind blowing with velocity $13\mathbf{i} + 5\mathbf{j}$ km per hour.

(a) Find the velocity vector the pilot should set so that the helicopter flies directly to A.

(b) Find the time taken for the journey.

(c) Find the velocity vector the pilot should set for a direct flight back to O. Assume that the wind blows with the same velocity throughout the flight back.

08 Components & Position Vectors III

Calculator Assumed

1. [6 marks: 1, 1, 1, 3]

   A particle P, initially at $5\mathbf{i} - 10\mathbf{j}$ metres, moves with velocity $3\mathbf{i} + 4\mathbf{j}$ ms^{-1}.

   (a) Find the position vector of P after 10 seconds.

   (b) Find the distance travelled by P after 10 seconds.

   (c) Find the position vector of P after t seconds.

   (d) When is P at a point with position vector $(65\mathbf{i} + 70\mathbf{j})$ metres.

Calculator Assumed

2. [9 marks: 3, 3, 3]

The position vector of particles A and B, t hours after 12 noon, are $\mathbf{r} = 12\mathbf{i} + 3\mathbf{j} + t(3\mathbf{i} + 4\mathbf{j})$ and $\mathbf{r} = -3\mathbf{i} - 5\mathbf{j} + t(2\mathbf{i} + 6\mathbf{j})$ metres respectively.

(a) Find in terms of t, the distance between A and B t hours after 12 noon.

(b) Find when A and B are 18 metres apart.

(c) Find when A is closest to B and find this distance.

Calculator Assumed

3. [9 marks: 2, 1, 2, 4]

The position vectors of A and B, t hours after 10 am are
$\mathbf{r} = -4\mathbf{i} - 4\mathbf{j} + t(2\mathbf{i} + 3\mathbf{j})$ and $\mathbf{r} = 3\mathbf{i} + 10\mathbf{j} + t(a\,\mathbf{i} + \mathbf{j})$ respectively.

(a) Find **AB** t hours after 10 am.

(b) Find in terms of a and t, the distance between A and B, t hours after 10 am.

(c) Explain why when collision between A and B occurs, $\mathbf{AB} = 0\mathbf{i} + 0\mathbf{j}$

(d) Find the value of a if the two particles never collide.

Calculator Assumed

4. [12 marks: 2, 4, 2, 4]

   At 9.00 am, boat A is located at $2\mathbf{i} + 3\mathbf{j}$ km and is travelling with velocity $2\mathbf{i} - 3\mathbf{j}$ km per hour. At 10.00 am, boat B is located at $2\mathbf{i} + 3\mathbf{j}$ km and is travelling with a speed of 5 km per hour. Let the velocity of boat B be $x\mathbf{i} + y\mathbf{j}$ km per hour.

   (a) Find the position vectors of A and B t hours after 10.00 am.

   (b) Show that when B intercepts A, $y = -1.5x$.

   (c) Hence, find the direction B should take in order to intercept A.

   (d) Find when B intercepts A.

Calculator Assumed

5. [6 marks: 1, 1, 2, 2]

Particle P starts moving from the point A with position vector $-2\mathbf{i} + 3\mathbf{j}$ metres with velocity $\mathbf{i} - 2\mathbf{j}$ metres per second. Particle Q starts moving from the point A at the same time with velocity $2\mathbf{i} + 3\mathbf{j}$ metres per second.

(a) Determine the position vector of P after t seconds.

(b) Determine the position vector of Q after t seconds.

(c) Find in terms of t, the distance between P and Q after t seconds.

(d) Use your answer in (c) to find when P and Q are 10 metres apart.

Calculator Assumed

6. [7 marks: 1, 2, 1, 3]

Particle P starts moving from the point with position vector $3\mathbf{i} + 5\mathbf{j}$ metres with velocity $2\mathbf{i} - 3\mathbf{j}$ metres per second.

(a) Determine the position vector of P after 3 seconds.

(b) Find the distance from P to the point with position vector $-2\mathbf{i} + \mathbf{j}$ after 3 seconds.

(c) Determine the position vector of P after t seconds.

(d) Find when P is closest to the origin and state this distance.

Calculator Assumed

7. [9 marks: 2, 2, 2, 3]

A speed boat is moving at a constant velocity of 40 kmh^{-1} in the direction with bearing 060°. Initially, the speed boat is $5\sqrt{2}$ km from a buoy and is in the direction with bearing 225° from the buoy. Given that **i** and **j** are the unit vectors in the Easterly direction and Northerly direction respectively, find:

(a) the initial position vector of the speed boat with respect to the buoy in terms of **i** and **j**.

(b) the direction vector of the speed boat in terms of **i** and **j**.

(c) the position vector of the speed boat with respect to the buoy t hours later.

(d) the time when the speed boat is nearest to the buoy and the least distance between the speed boat and the buoy.

Calculator Assumed

8. [9 marks: 4, 2, 3]

Yacht A starts sailing from the point L with position vector $5\mathbf{i} - 2\mathbf{j}$ metres with velocity $-\mathbf{i} + 3\mathbf{j}$. Yacht B starts sailing 10 seconds later with velocity $2\mathbf{i} + \mathbf{j}$ metres per second, from the point M with position vector $4\mathbf{i} - 3\mathbf{j}$ metres per second. t is time in seconds from the moment B starts sailing.

(a) Find in terms of t, the distance between A and B after t seconds.

(b) When will A and B be 40 metres apart?

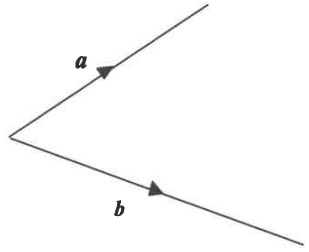
(c) When will the two yachts be closest together? State this distance.

09 Relative Displacement

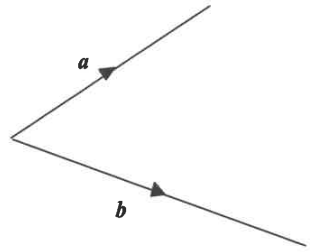
Calculator Free

1. [4 marks: 1, 1, 1, 1]

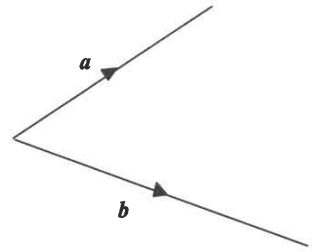
   (a) Indicate clearly in the diagram below $\boldsymbol{a} + \boldsymbol{b}$.



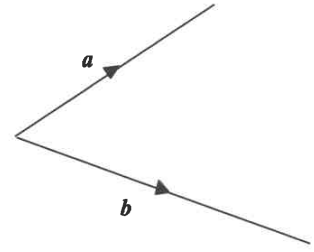
   (b) Indicate clearly in the diagram below $\boldsymbol{a} - \boldsymbol{b}$.



   (c) Indicate clearly in the diagram below the vector $\boldsymbol{b}$ relative to $\boldsymbol{a}$.



   (d) Indicate clearly in the diagram below the vector $\boldsymbol{a}$ relative to $\boldsymbol{b}$.



Calculator Free

2. [4 marks: 1, 1, 2]

The position vectors of the points A, B, and C with respect to the origin O are $\mathbf{i} + \mathbf{j}$, $2\mathbf{i} - \mathbf{j}$ and $-4\mathbf{i} + 5\mathbf{j}$ respectively.

(a) Find the position vector of A relative to B.

(b) Find the displacement of C relative to A.

(c) The position vector of the point D relative to C is $-2\mathbf{i} - 3\mathbf{j}$.
Find the position vector of D relative to O.

---

3. [2 marks: 1, 1]

The position vectors of the yachts WAone and WAtwo are $20\mathbf{i} + 40\mathbf{j}$ km and $-10\mathbf{i} + 50\mathbf{j}$ km respectively.

(a) Find the position vector of WAtwo relative to WAone.

(b) Hence, find the distance between the two yachts.

Calculator Assumed

4. [5 marks: 2, 1, 2]

The position vector of Peter relative to a flag pole is $20\mathbf{i} + 40\mathbf{j}$ metres. Relative to Peter, Joe has position vector $5\mathbf{i} - 15\mathbf{j}$ metres.

(a) Find the position vector of Joe relative to the flagpole.

(b) Hence, find the distance between Joe and the flag pole.

(c) The position vector of Kelly relative to Joe is $a\mathbf{i} + 20\mathbf{j}$ metres.
Find the value of a if the distance between Kelly and Joe is 50 metres.

---

5. [4 marks]

Vectortown has position vector $-20\mathbf{i} + 10\mathbf{j}$ km. The position vector of Trigtown relative to Vectortown is $-40\mathbf{i} - 15\mathbf{j}$ km. The position vector of Easytown relative to Trigtown is $4\mathbf{i} + 70\mathbf{j}$ km. Find the position vector of Easytown.

10 Relative Velocity

Calculator Assumed

1. [7 marks: 1, 1, 3, 2]

   Relative to an observer at O, A is moving with velocity $6\mathbf{i} + 9\mathbf{j}$ ms^{-1} and B is moving with velocity $-3\mathbf{i} + 4\mathbf{j}$ ms^{-1}.
   (a) Find the velocity of A relative to B

   (b) What is the speed of A relative to B?

   (c) The velocity of C relative to B is $4\mathbf{i} - 5\mathbf{j}$ ms^{-1}. Find the velocity of C relative to A.

   (d) In what direction is C moving relative to A?

Calculator Assumed

2. [7 marks: 3, 4]

L is travelling along bearing 060° with a speed of 10 kmh^{-1}. M is travelling along bearing 210° with a speed of 5 kmh^{-1}.

(a) Draw a clearly labelled vector diagram indicating the velocity vector of L relative to M.

(b) Use trigonometry to find the speed and direction of L relative to M.

Calculator Assumed

3. [7 marks: 2, 2, 1, 2]

P is travelling along bearing 045° with a speed of 20 kmh^{-1}. Q is travelling along bearing 240° with a speed of 15 kmh^{-1}. **i** and **j** are unit vectors in the Easterly and Northerly directions respectively.

(a) Express the velocities of P and Q in terms of **i** and **j**.

(b) Find the velocity of Q relative to P.

(c) What is the speed of Q relative to P?

(d) What is the direction of Q relative to P?

11 Relative Vectors

Calculator Assumed

1. [5 marks: 2, 3]

   James is running along bearing 050° with a speed of 5 ms^{-1}. Wesley is running along bearing 300° with speed 4 ms^{-1}. **i** and **j** are unit vectors in the Easterly and Northerly directions respectively.

   (a) Find in component form the velocity of Wesley relative to James.

   (b) Find how fast and in what direction is Wesley moving away from James.

Calculator Assumed

2. [6 marks: 2, 4]

A yacht is sailing on a bearing of 050° with speed 12 kmh^{-1}. Sarah on the yacht measures the wind as blowing with a speed of 10 kmh^{-1} from a bearing of 300°.

(a) Sketch a clearly labelled velocity vector diagram that shows the relationship between the velocity of the yacht, the true velocity of the wind and the velocity of wind relative to the yacht.

(b) Find the true speed and direction of the wind.

Calculator Assumed

3. [7 marks: 2, 2, 3]

A yacht Y is moving with velocity $2\mathbf{i} + 5\mathbf{j}$ kmh^{-1}. A sailor on board the yacht measures the wind as blowing with velocity $-3\mathbf{i} - 2\mathbf{j}$ kmh^{-1}.

(a) Find the velocity of the wind.

To a sailor on a second yacht Z, the wind appears to be blowing with velocity $2\mathbf{i} + 4\mathbf{j}$ kmh^{-1}.

(b) Find the velocity of the second yacht.

(c) How fast is yacht Z moving away from yacht Y and in what direction?

Calculator Assumed

4. [4 marks]

A ship is travelling with a speed of 20 knots along bearing 080°. Relative to the ship, the wind is blowing from 310° with a speed of 8 knots. Let **i** and **j** be unit vectors in the Easterly and Northerly directions respectively. By expressing the given velocities in component form, find the true speed and direction of the wind.

Calculator Assumed

5. [11 marks: 2, 2, 7]

May is running along 025° at 4 kmh^{-1}.
Relative to May, Fay is running with speed a kmh^{-1} along bearing 150°.

(a) Find in terms of a, the velocity of Fay, v_f.

Jane is running due East at 2 kmh^{-1}.
Relative to Jane, Fay is running with speed b kmh^{-1} along bearing 120°.

(b) Find in terms of b, the velocity of Fay, v_f.

(c) Use your answers in (a) & (b) to find a and b. Hence, find the true speed and direction with which Fay is running.

Calculator Assumed

6. [9 marks: 3, 3, 3]

Tom is in a hot-air balloon travelling with velocity $3\mathbf{i} + 4\mathbf{j}$ km and to Tom the wind is blowing along bearing 045°. Jerry is in another hot-air balloon flying at the same altitude and travelling with velocity $-4\mathbf{i} + \mathbf{j}$. To Jerry, the wind is blowing along bearing 60°. Let the true velocity of the wind be $x\mathbf{i} + y\mathbf{j}$ kmh^{-1}. $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the Easterly and Northerly directions respectively.

(a) Find in terms of x and y, the velocity of the wind relative to Tom. Hence, show that $x - y = -1$.

(b) Find in terms of x and y, the velocity of the wind relative to Jerry. Hence, show that $x - \sqrt{3}y = -4 - \sqrt{3}$.

(c) Find the true velocity of the wind.

Calculator Assumed

7. [4 marks: 1, 1, 2]

   At 1000 hours, the position vectors of A and B are $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively. A is moving with velocity $\boldsymbol{u}$ and B is moving with velocity $\boldsymbol{v}$.

   (a) Find ${}_{B}\boldsymbol{r}_{A}$ the position vector of B relative to A.

   (b) Find ${}_{A}\boldsymbol{v}_{B}$ the velocity of A relative to B.

   (c) Show that if A collides with B after t seconds, ${}_{B}\boldsymbol{r}_{A} = t\,{}_{A}\boldsymbol{v}_{B}$ where $t > 0$.

---

8. [4 marks: 3, 1]

   At 0800 hours, the position vector of Q relative to P is $45\mathbf{i} - 60\mathbf{j}$ km. The velocity of P relative to Q is a constant $9\mathbf{i} + m\,\mathbf{j}$ kmh^{-1}.

   (a) Find the value of m, if P intercepts Q.

   (b) Find when P intercepts Q.

Calculator Assumed

9. [6 marks: 1, 1, 4]

At 0900 hours, P is located at $20\mathbf{i} - 10\mathbf{j}$ km and is travelling with a constant velocity of $-4\mathbf{i} + 6\mathbf{j}$ kmh^{-1}. At the same time, Q is located at $50\mathbf{i} + 40\mathbf{j}$ km and is travelling with a constant velocity of $-10\mathbf{i} - 4\mathbf{j}$ kmh^{-1}.

(a) Find the displacement of Q relative to P at 0900 hours.

(b) Find the velocity of P relative to Q at 0900 hours.

(c) Use your answers in (a) and (b) to determine when and where P will collide with Q.