Calculator Free

1. [8 marks: 2, 1, 1, 4]

   Given that $\boldsymbol{a} = 3\,\mathbf{i} + 10\,\mathbf{j} + 8\,\mathbf{k}$ and $\boldsymbol{b} = 2\,\mathbf{j} + 3\,\mathbf{k}$, find:

   (a) $|\boldsymbol{a} - 2\boldsymbol{b}|$

   (b) the unit vector parallel to $\boldsymbol{a} - 2\boldsymbol{b}$

   (c) a vector that is parallel to $\boldsymbol{a} - 2\boldsymbol{b}$ but with a magnitude of 10

   (d) $\boldsymbol{a}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$ where $\boldsymbol{p} = -\mathbf{i} + 4\,\mathbf{k}$ and $\boldsymbol{q} = 2\mathbf{i} + 5\,\mathbf{j} + 2\mathbf{k}$.

---

2. [6 marks]

   $\mathbf{OA} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{OB} = 2\mathbf{i} + b\,\mathbf{j} - \mathbf{k}$ and $\mathbf{OC} = 6\mathbf{i} + 3\mathbf{j} + c\,\mathbf{k}$.
   Find b and c if A, B and C are collinear.

Calculator Free

3. [6 marks]

Vector $\alpha\mathbf{i} + \beta\mathbf{j} + \sqrt{2\alpha\beta}\,\mathbf{k}$ has a magnitude of 10 and is parallel to vector $3\mathbf{i} + 4\mathbf{j} + 2\sqrt{6}\mathbf{k}$. Find all possible values of α and β.

---

4. [3 marks]

The points P and Q have position vectors $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $-\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ respectively. The point K is such that $\mathbf{PQ} = -2\mathbf{QK}$.
Find the position vector of K.

---

5. [6 marks: 2, 2, 2]

Given that $\boldsymbol{a} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$, find:

(a) $(\boldsymbol{a} - \boldsymbol{b}) \bullet \boldsymbol{c}$

(b) $(\boldsymbol{a} - \boldsymbol{b}) \bullet (\boldsymbol{a} - \boldsymbol{c})$

(c) m if $\boldsymbol{b} \bullet (2\mathbf{i} - m\mathbf{j} + 3\mathbf{k}) = -1$

Calculator Free

6. [5 marks: 3, 1, 1]

   (a) Find the acute angle between the vectors 2**i** + **j** + 2**k** and **i** + **j**.

   Hence, or otherwise, find the acute angle between:

   (b) 2**i** + **j** + 2**k** and –**i** – **j**

   (c) –4**i** – 2**j** – 4**k** and –5**i** – 5**j.**

---

7. [4 marks]

   Find the value(s) of α if the angle between the vectors **i** + **k** and 2 **i** + α **j** is 60°.

---

8. [6 marks]

   u = *a* **i** – **j** + 3**k** has the same magnitude as ***v*** = (7 – *b*) **i** + (*a* + *c*) **j** + (4*b* – *c*) **k**.
   Find the values of *a*, *b* and *c* if ***u*** and ***v*** act in opposite directions.

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Calculator Free

9. [6 marks: 3, 3]

(a) Find a unit vector that is parallel to $-\mathbf{i} + \mathbf{j} - \mathbf{k}$ and perpendicular to $-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

(b) Find a unit vector that is perpendicular to both $-\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

---

10. [8 marks: 3, 1, 4]

Let $\boldsymbol{u} = < 2, 1, 2 >$, $\boldsymbol{v} = < 1, 0, 1>$ and $\boldsymbol{w} = < -1, 1, 0 >$.

(a) Use the result $|\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}|\,|\boldsymbol{b}|\sin\theta$, where θ is the acute angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ to calculate the sine of the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$.

(b) Determine the area of the parallelogram with sides parallel to $\boldsymbol{v}$ and $\boldsymbol{w}$.

(c) The scalar projection of vector $\boldsymbol{a}$ onto vector $\boldsymbol{b}$ is given by $\boldsymbol{a} \bullet \hat{\boldsymbol{b}}$.
Find the vector projection of $\boldsymbol{u}$ onto $\boldsymbol{v} \times \boldsymbol{w}$.

Calculator Assumed

11. [4 marks]

Prove that the line segments congruent to the vectors $\begin{pmatrix} 3 \\ 1 \\ 11 \end{pmatrix}$, $\begin{pmatrix} -6 \\ -14 \\ -9 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -13 \\ 2 \end{pmatrix}$ form a right angled triangle.

12. [6 marks]

The sides of an equilateral triangle are congruent with the vectors

$\boldsymbol{u} = \begin{pmatrix} \frac{-1}{2} \\ 0 \\ \frac{-\sqrt{3}}{2} \end{pmatrix}$, $\boldsymbol{v} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{-\sqrt{3}}{2} \end{pmatrix}$ and $\boldsymbol{w}$. Find $\boldsymbol{w}$.

Calculator Free

1. [9 marks: 3, 3, 3]

   The points A and B have position vectors $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ respectively.

   (a) Find the position vector of the point K which is 6 units from A in the direction $-2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

   (b) Find the position vector of the point L which is 21 units from A in the direction AB.

   (c) Find the area of ΔKAL. Show clearly how you obtained your answer.

Calculator Free

2. [10 marks: 3, 3, 4]

   Let $\boldsymbol{u} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\boldsymbol{v} = -\mathbf{i} + \mathbf{k}$ and $\boldsymbol{w} = \langle 2, -2, 2 \rangle$.

   (a) Find the acute angle between $\boldsymbol{u}$ and $\boldsymbol{v}$.

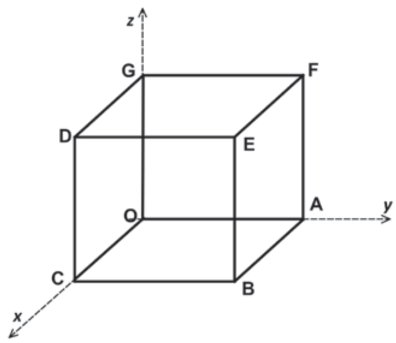
   (b) Find the area of a triangle with sides parallel and congruent to $\boldsymbol{u}$ and $\boldsymbol{v}$.

   (c) A triangular pyramid has sides parallel and congruent to $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$. Determine the volume of this triangular pyramid.

Calculator Assumed

3. [11 marks: 2, 2, 4, 3]

A rectangular box OABCDEFG rests on the x-y plane as shown. The vertex O has position vector < 0, 0, 0 > cm. CB = 10 cm, AB = BE = 8 cm.



(a) State the position vectors of the vertices C and E of this box.

(b) Use vector methods to find the length of the diagonal BG.

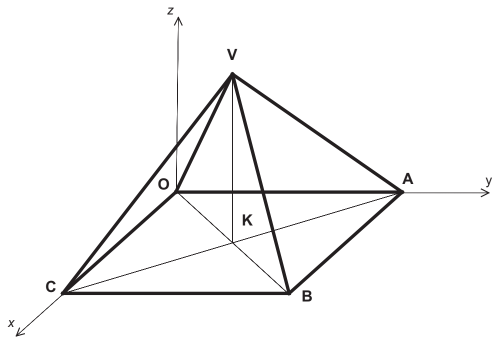
(c) Use a cross-product method to calculate exactly sin(∠GBF).

(d) Calculate the exact area of ΔBGF.

Calculator Assumed

4. [10 marks: 2, 8]

A regular pyramid VOABC rests on the x-y plane as shown. The vertices O, A, and C have position vectors < 0, 0, 0 >, < 0, 4, 0 > and < 6, 0, 0 > cm respectively. The vertex V is 5 cm vertically above the rectangular base OABC.



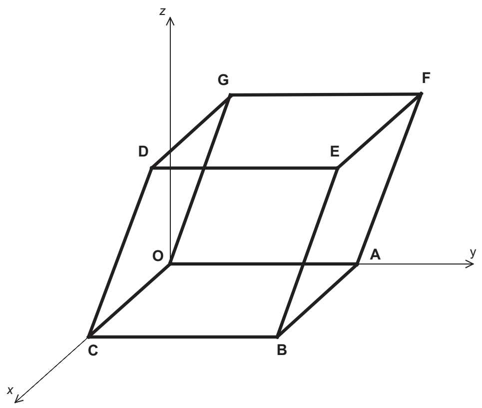
(a) State the position vector of V.

(b) Use a vector method to calculate the exact total surface area of this pyramid.

Calculator Assumed

5. [11 marks: 4, 3, 2, 2]

A parallelepiped OABCDEFG rests on the x-y plane as shown. [All six faces are parallelograms with opposite faces congruent.] The vertices O, A, C and G have position vectors < 0, 0, 0 >, < 0, 3, 0 >, < 2, 0, 0 > and < 0, 1, 2 > cm respectively.



(a) State the position vectors of the vertices D and E of this box.

(b) Determine the angle between **GE** and **AB**.

(c) The volume of the parallelepiped = Area of base × Perpendicular height.

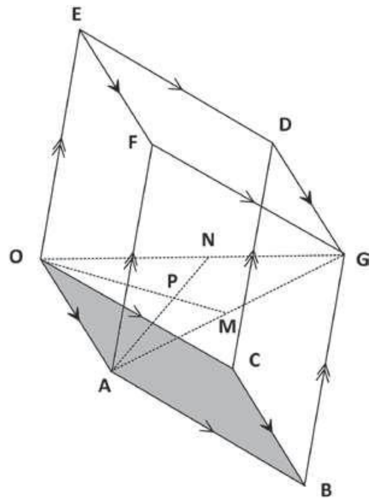
(i) Find the area of the base OABC.

(ii) Find the volume of the parallelepiped.

Calculator Assumed

6. [13 marks: 2, 4, 2, 5]

The given diagram shows a parallelepiped OABCDEFG. The opposite faces of the solid are congruent parallelograms which are parallel to each other. **OA** = $\boldsymbol{a}$, **OC** = $\boldsymbol{c}$ and **OE** = $\boldsymbol{e}$. The point M is the midpoint of AG and the point N is the midpoint of OG. OM and AN intersect at P.
Let **OP** = α**OM** and **AP** = β**AN**.



(a) Express **AG** and **OG** in terms of $\boldsymbol{a}$, $\boldsymbol{c}$ and/or $\boldsymbol{e}$.

(b) Express **OM** and **AN** in terms of $\boldsymbol{a}$, $\boldsymbol{c}$ and/or $\boldsymbol{e}$.

(c) Express **OP** and **AP** in terms of $\boldsymbol{a}$, $\boldsymbol{c}$ and/or $\boldsymbol{e}$.

(d) Determine the values of α and β.

Calculator Free

1. [6 marks: 3, 3]

The vector equation of a line is given by $\boldsymbol{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(a) Determine with reasons if the point (–1, 4, –6) lies on the line.

(b) The point with position vector $< -2k, k, k + 5 >$ lies on this line, find the value(s) of k.

---

2. [7 marks: 2, 2, 3]

The parametric equation of a line is given by $x = \frac{1 + \lambda}{2}$, $y = \frac{-1 + 2\lambda}{2}$, $z = \frac{4 - 3\lambda}{5}$.

(a) Determine the Cartesian equation of this line.

(b) Find the vector equation of this line.

(c) The points A, B and C on this line are such that $\lambda = -1$, $\lambda = 0$ and $\lambda = 1$ respectively. Find the ratio with which the point B divides the line AC.

Calculator Assumed

3. [8 marks: 4, 4]

The point A $(a, -a, 0)$ lies on the line L with equation $\boldsymbol{r} = \begin{pmatrix} b \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ b \\ -1 \end{pmatrix}$.

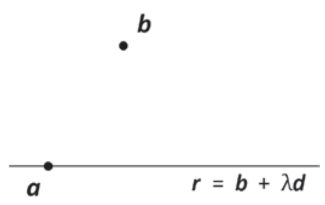
(a) Find the value(s) of a and b.

(b) Use a vector method to find the shortest distance between the point P (1, 0, 3) and the line L.

---

4. [8 marks: 3, 5]

(a) Use $|\boldsymbol{u} \times \boldsymbol{v}| = |\boldsymbol{u}|\,|\boldsymbol{v}| \sin\theta$ where θ is the acute angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ to prove that the shortest distance between the point with position vector $\boldsymbol{b}$ to the line with equation $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{d}$ is given by $|(\boldsymbol{b} - \boldsymbol{a}) \times \hat{\boldsymbol{d}}|$.



Calculator Assumed

4. (b) ABCD is a trapezium. The points A, B, and D have position vectors < 1, 0 1 >, < 0, 2, 0 > and < –1, 1 1 > respectively. The point C is such that **BC** is parallel to **AD** and has magnitude 3 times that of **AD**.
Calculate the area of the trapezium ABCD.

---

5. [9 marks: 2, 4, 3]

The lines L1 and L2 respectively have equations $\boldsymbol{r}$ = < 1, 1, 2 > + λ < –2, 1, –3 > and $\boldsymbol{r}$ = < 2, 1, 4 > + μ < 1, –1, 1 >. The lines L1 and L2 intersect at K.

(a) Find the acute angle between L1 and L2.

(b) Use a vector method to find the position vector of K.

(c) Find the vector equation of the line passing through the point K and perpendicular to L1 and L2.

Calculator Assumed

6. [11 marks: 3, 1, 1, 4, 2]

The lines L1 and L2 have equations $\boldsymbol{r} = < 0, 1, -1 > + \lambda < 1, 1, 1 >$ and $\boldsymbol{r} = < 1, 0, -2 > + \mu < 1, 2, \alpha >$ respectively.

(a) Find the value(s) of α if the lines L1 and L2 are non-intersecting.

(b) Let $\alpha = 1$. The points P and Q are points respectively on lines L1 and L2.

(i) Determine the position vectors of P and Q in terms of λ and/or μ.

(ii) Find **PQ** in terms of λ and/or μ.

(iii) Find λ and μ if **PQ** is perpendicular to both L1 and L2.

(iv) Hence, find the shortest distance between L1 and L2.

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Calculator Assumed

7. [10 marks: 4, 2, 2, 2]

The line L1 has equation $\boldsymbol{r} = < -4 + \lambda, -2\lambda, \lambda - 2 >$. The point B $(-4, 0, -2)$ lies on the line L1. The point A has position vector $< 2, -2, 0 >$.

(a) Find the position vector of the point K on the line L1 such that AK is perpendicular to L1.

(b) Hence, find the shortest distance between A and the line L1.

(c) Find the size of $\angle$BAK.

(d) Find the area of ΔAKB.

---

8. [11 marks: 4, 1, 4, 2]

The point A with position vector $< 1 + m, -m, -1 >$ lies on line L1 with equation $\boldsymbol{r} = < 1, 0, -1 > + \lambda < 1, -1, 0 >$. The point B with position vector $< 2 - 2n, 1 + n, n >$ lies on line L2 with equation $\boldsymbol{r} = < 2, 1, 0 > + \mu < -2, 1, 1 >$.

(a) Show that L1 and L2 are skew lines (non-intersecting and non-parallel).

Calculator Assumed

8. (b) Find **BA** in terms of m and n.

(c) Find the value(s) of m and n if **BA** is perpendicular to both L1 and L2.

(d) Hence, find the shortest distance between the lines L1 and L2.

---

9. [9 marks: 4, 4, 1]

The point A (1, 0, –2) lies on the line L1 which is parallel to $\boldsymbol{d_1}$ = < 4, –2, –4 >.
The point B (6, –1, –10) lies on the line L2 which is parallel to $\boldsymbol{d_2}$ = < 2, 1, 2 >.

(a) Prove that the lines L1 and L2 are neither parallel nor intersecting.

(b) Determine the scalar projection of **BA** onto $\boldsymbol{d_1} \times \boldsymbol{d_2}$.

(c) Hence, find the shortest distance between the lines L1 and L2.

Calculator Assumed

10. [12 marks: 4, 4, 4]

A tunnel in the form of a straight line is bored simultaneously from its two ends A and B. The ends A and B have position vectors < –4, 13, 2 > *m* and < –19, –17, –3 > *m* respectively. The tunnel from A is bored in the direction < 3, 2, –1 > while the tunnel from B is bored in the direction < 2, 3, u >.

(a) Determine the value(s) of u so that the two sections will meet.

(b) For the value(s) of u in (a), state the difference in the length(s) between the two sections.

(c) For u = 0.2, determine the closest the two sections get to each other.

Calculator Assumed

11. [11 marks: 2, 5, 4]

The line *L* passes through the points P and Q with position vectors < 2, –4, 6 > and < 12, 1, 11 > respectively. The point K has position vector < 4, –8, 12 > and the point S is the reflection of the point K about the line L.

(a) Determine the vector equation of the line *L*.

(b) Determine the position vector for S.
[Hint: Determine the point R on line *L* that is closest to the point K.]

(c) Determine the area of the quadrilateral KPSQ.

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Calculator Assumed

12. [8 marks: 2, 4, 4]

(a) Let $\boldsymbol{r}$ be the position vector of a variable point, $\boldsymbol{a}$ be the position vector of a fixed point and $\boldsymbol{d}$ be a constant free vector. Explain why $(\boldsymbol{r} - \boldsymbol{a}) \times \boldsymbol{d} = \boldsymbol{0}$ represents the vector equation of a line.

(b) The line L_1 has equation $\boldsymbol{r} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$.

Rewrite the equation of this line in the form $\boldsymbol{r} \times \boldsymbol{a} = \boldsymbol{b}$.

(c) The line L_2 has equation $\boldsymbol{r} \times \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ 6 \end{pmatrix}$.

Rewrite the equation of this line in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{d}$.

Calculator Free

1. [7 marks: 2, 5] [TISC]

The line L with equation $\boldsymbol{r} = 2\boldsymbol{i} + 2\lambda\boldsymbol{j} + (1 - \lambda)\boldsymbol{k}$ is perpendicular to the plane P.

(a) The point A with position vector $-\boldsymbol{i} + \boldsymbol{k}$ lies on the plane P.
Determine the vector equation of the plane P.

(b) B is a point on line L. The distance between A and B is $\sqrt{14}$ units.
Determine the position vector of the point B.

---

2. [7 marks: 3, 4]

The points with position vectors $< 1, 0, 2 >$, $< -1, 1, 0 >$ and $< 0, 1, -1 >$ lie on a plane.

(a) Determine the vector equation of the plane in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$.

(b) Determine the Cartesian equation of the plane.

Calculator Free

3. [8 marks: 3, 5]

The plane P has equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

(a) Determine with reasons if the point A with position vector $2\boldsymbol{i} - 5\boldsymbol{j} + \boldsymbol{k}$ lies on the plane P

(b) Plane Q has equation $2x - y + z = 1$.
Calculate the acute angle between planes P and Q.

Calculator Assumed

4. [9 marks: 1, 1, 2, 2, 3]

The points A and B have position vectors < 1, –2, 1 > and < 0, 0, 1 > respectively. The plane Π has equation $\boldsymbol{r} \bullet <2, -1, 1> = 1$.

(a) Find a unit vector perpendicular to the plane Π.

(b) Show that the point B lies on the plane Π.

(c) Find $|\mathbf{BA}|$.

(d) Find the acute angle between **BA** and the normal to the plane Π.

(e) Hence, find the minimum distance between the point A and the plane Π.

Calculator Assumed

5. [4 marks]

Find the value(s) of m if the line $\boldsymbol{r} = \langle 2 + m\lambda, -3, 1 + \lambda \rangle$ is inclined to the plane $\boldsymbol{r} \bullet \langle 0, -1, 1 \rangle = 10$ at an angle of 30°.

---

6. [6 marks: 1, 1, 1, 1, 2]

The planes Π_1 and Π_2 have equations $\boldsymbol{r} \bullet \langle 1, -1, 2 \rangle = 5$ and $\boldsymbol{r} \bullet \langle 1, -1, 2 \rangle = 8$ respectively.

(a) Show that the two given planes are parallel.

(b) Find a unit vector normal to these two given planes.

(c) The points A and B with position vectors $\langle 0, -5, 0 \rangle$ and $\langle 0, -8, 0 \rangle$ lie on Π_1 and Π_2 respectively. Find $|\mathbf{AB}|$.

(d) Find the acute angle between **AB** and the normal to the two given planes.

(e) Hence, find the distance between the two given planes.

Calculator Assumed

7. [11 marks: 2, 3, 3, 3]

The plane P has equation $\boldsymbol{r} \bullet (2\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}) = -2$.

The line L has equation $\dfrac{x}{-1} = \dfrac{y-2}{1} = \dfrac{z+1}{2}$.

(a) Find the vector equation for line L

(b) Show that the line L does not intersect the plane P

(c) Find the Cartesian equation of the plane Q containing line L and parallel to P.

(d) Determine with reasons if planes P and Q are on the same side or on opposite sides of the origin (0, 0, 0).

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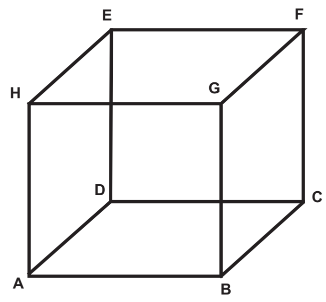
Calculator Assumed

8. [13 marks: 4, 2, 2, 3, 2]

ABCDEFGH is a rectangular prism. The vertices A, B, C and G have position vectors $< 2, 2, 2 >$, $< 1, 1, 1 >$, $< 2, 1, 0 >$ and $< 0, 3, 0 >$ respectively.



(a) Find the position vectors of the vertices D and E.

(b) Find the vector equation of the edge AB.

(c) Find the vector equation of the plane ABCD in the form $\boldsymbol{r} \bullet \boldsymbol{n} = \rho$.

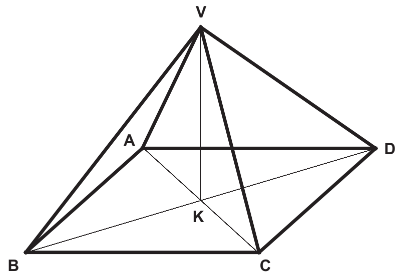
(d) Use vectors to find the acute angle between the planes ADFG and ABCD.

(e) Find the volume of the rectangular prism.

Calculator Assumed

9. [11 marks: 2, 2, 2, 3, 2] [TISC]

The accompanying diagram shows a rectangular pyramid VABCD. The vertices B, C and D have position vectors **i** + **j** + **k**, 3**j** + 3**k** and 2**i** + 2**j** + 5**k** respectively. The vertex V with position vector $-\frac{1}{2}\mathbf{i} - \frac{1}{2}\mathbf{j} + 4\mathbf{k}$ is vertically above K, the centre of the rectangular base ABCD.



(a) Use vectors to find the position vector of the vertex A.

(b) Find **KV**.

(c) Find the vector equation of the line passing through C and V.

(d) Find the vector equation of the plane ABCD in the form $\boldsymbol{r} \bullet \boldsymbol{n} = \rho$.

(e) Find the acute angle between the line VC and the plane ABCD.

Calculator Assumed

10. [10 marks: 4, 4, 2]

A tetrahedron has vertices V, A, B and C with position vectors < –1, –1, 9 >, < –2, 2, 4 >, < 0, –4, 4 > and < 0, –1, 2 > respectively.

(a) Find the area of the base ABC.

(b) Find the shortest distance between the vertex V and the base ABC.

(c) Hence, determine the volume of the tetrahedron VABC.

---

11. [11 marks: 2, 3, 3, 3]

The vertices A, B and C of a parallelogram ABCD respectively have position vectors < 1, 2, 1 >, <–1, 2, 0 >, < 1, –2, –3 >.

(a) Determine the position vector of the vertex D.

Calculator Assumed

11. (b) Determine the vector equation of the plane containing ABCD.

The point V has position vector < 3, 4, –5 >.

(c) Prove that V lies on the perpendicular through the midpoint of ABCD.

(d) Determine the volume of the pyramid VABCD.

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Calculator Assumed

12. [11 marks: 5, 6]

An object A is acted on by three forces $12\boldsymbol{i} + 10\boldsymbol{j} + 5\boldsymbol{k}$ N, $-5\boldsymbol{i} + 8\boldsymbol{j} - 10\boldsymbol{k}$ N and $6\boldsymbol{i} - 8\boldsymbol{j} + 4\boldsymbol{k}$ N.

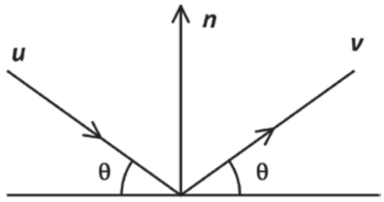
(a) Determine the magnitude of the resultant force and the acute angle it makes with the x-y plane.

(b) A fourth force $\boldsymbol{F}$ of magnitude $\sqrt{306}$ N acts on A so that the resultant now acts in the same direction as the vector $2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$. Determine $\boldsymbol{F}$.

Calculator Assumed

13. [11 marks: 2, 2, 2, 2, 3]

A ball travelling with velocity $\boldsymbol{u}$ = < 2, 1, –2 > ms^{-1}, hits the surface of a plane P with equation $\boldsymbol{r} \bullet$ < 2, –2, –1 > = 10 at an angle of θ° with the plane. It rebounds with velocity $\boldsymbol{v}$ at an angle of θ° with the plane. $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{n}$ the normal to the plane P all lie on the same plane.



(a) Determine the unit vector normal to the plane.

(b) Determine the exact value for cos(90° – θ°).

(c) Determine $\boldsymbol{a}$, the component of $\boldsymbol{u}$ that is perpendicular to the plane P.

(d) Determine $\boldsymbol{b}$, the component of $\boldsymbol{u}$ that is parallel to the plane P.

(e) Determine $\boldsymbol{v}$ if $|\boldsymbol{v}| = 0.9|\boldsymbol{u}|$.

Calculator Assumed

14. [12 marks: 3, 3, 4, 2]

Plane P_1 has equation $x + 2y - 2z = 11$.

(a) Determine with reasons if the line L with equation $\frac{1-x}{2} = \frac{y-3}{2} = z + 2$ lies in the plane P_1.

The plane P_2 contains the line L and is perpendicular to P_1.

(b) Determine the equation of the plane P_2.

Calculator Assumed

14. The plane P_3 is perpendicular to both P_1 and P_2. The shortest distance from the point A with position vector < 1, 3, –2 > to the plane P_3 is 6 units.

(c) Determine the equation of the plane P_3.

(d) Determine the points of intersection(s) of the planes P_1, P_2 and P_3.

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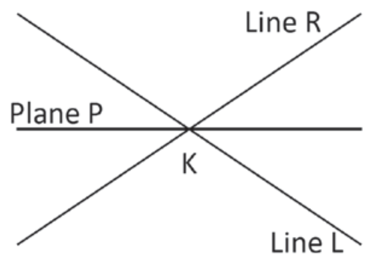
Calculator Assumed

15. [7 marks: 3, 4]

The line L with equation $\boldsymbol{r} = \begin{pmatrix}1\\1\\3\end{pmatrix} + \lambda\begin{pmatrix}0\\1\\2\end{pmatrix}$ meets the plane P with equation $2x + y + z = 12$ at the point K.



(a) Determine the position vector of K.

The line R is the reflection of line L about the plane P.

(b) Determine the vector equation of line R.

Calculator Free

1. [10 marks: 2, 2, 2, 4]

The sphere S has centre (1, –1, 2) and radius 5. The point P with coordinates (5, 2, 2) lies on this sphere.

(a) Determine the vector and Cartesian equations of this sphere.

(b) PQ is a diameter of this sphere, determine the coordinates of Q.

(c) The sphere intersects the y-z plane in the form of a circle.
Determine the Cartesian equation of this circle.

(d) Determine the value(s) of k if the point with position vector $< 1, 3, k >$ is outside this sphere.

Calculator Assumed

2. [8 marks: 4, 4]

The sphere S has Cartesian equation $9x^2 + 9y^2 + 9z^2 - 6x + 54y + 18z + 10 = 0$.

(a) Determine the position vector of the centre of this sphere and its radius.

(b) Determine with reasons if the plane with equation $\boldsymbol{r} \bullet < 2, 1, 2 > = 2$ will intersect this sphere.

---

3. [10 marks: 4, 6]

The line with equation $x - 1 = \dfrac{y-2}{2} = 1 - z$ intersects the sphere with equation $x^2 + y^2 + z^2 + 2x + 4y - 2z - 30 = 0$ at the points A and B.

(a) Determine the vector equations of the line and the sphere.

Calculator Assumed

3. (b) Hence, or otherwise, determine the exact length of the chord AB.

---

4. [9 marks: 5, 4]

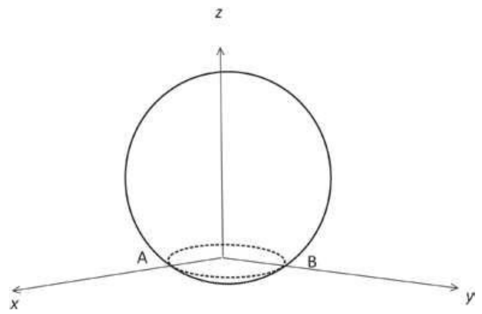
(a) The point A has position vector < 2, 1, –2 >. The plane P has equation $\boldsymbol{r} \bullet \langle 1, 2, 2 \rangle = 1$. The sphere S has radius 6. The plane P is tangential to the sphere S at the point A. Determine the vector equation of the sphere S.

(b) The line L has equation $\dfrac{x-1}{2} = \dfrac{y+1}{-1} = z-2 = \lambda$. A sphere S has equation $(x-1)^2 + (y+1)^2 + (z-2)^2 = 6$. Determine the point(s) of intersection between the line L and the sphere S.

Calculator Assumed

5. [6 marks: 1, 2, 3]

The sphere shown in the given diagram intersects the x-y plane. The circle of intersection cuts the x-axis and y-axis at the points A and B respectively and has vector equation $|\boldsymbol{r}| = 3$.



(a) Express the equation of the circle of intersection in Cartesian form.

(b) Explain clearly why C, the centre of the sphere, must lie on the z-axis.

(c) The sphere has a radius of 5 units. Determine a possible vector equation of the sphere.

---

6. [8 marks: 4, 4]

Consider the sphere S with equation $|\boldsymbol{r} - \langle 1, 1, 1 \rangle| = 3$.

(a) Prove that the line L with equation $\boldsymbol{r} = \langle 3, -1, 2 \rangle + \lambda \langle 1, 1, 0 \rangle$ is a tangent to the sphere S.

Calculator Assumed

6. (b) Prove that the plane with equation $\boldsymbol{r} \bullet < 2, 2, 1 > = 14$ is tangential to S.

---

7. [7 marks: 4, 3]

Consider the sphere S with equation $\mid \boldsymbol{r} - < 1, 0, 1 > \mid = 2$.

(a) Determine the equation of the curve of intersection of S with the x-y plane. Describe this curve.

(b) Determine the equation of the curve of intersection of S with the sphere T with equation $\mid \boldsymbol{r} - < 1, 0, 2 > \mid = \sqrt{3}$. Describe this curve.

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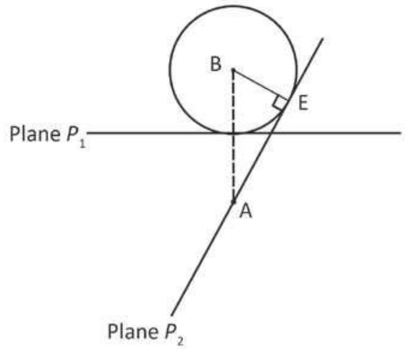
Calculator Assumed

8. [10 marks: 7, 3]

Plane P_1 has equation $2x - 3y + 6z = 21$. The point A has position vector $< -15, 12, -10 >$. Plane P_1 is tangential to sphere S_1. B, the centre of S_1 is the reflection of the point A about P_1. The plane P_2 passes through A and is tangential to the sphere S_1 at E.



(a) Determine the vector equation of S_1.

(b) Determine the angle between the planes P_1 and P_2.

Calculator Assumed

1. [5 marks]

The position vectors of particles A and B at time t seconds are respectively
$\boldsymbol{r}_A = \begin{pmatrix} 2+t \\ 3-t \\ -2+4t \end{pmatrix}$ and $\boldsymbol{r}_B = \begin{pmatrix} -4+3t \\ 4-2t \\ -1+7t \end{pmatrix}$. Find when and where A crosses the path of B.

---

2. [11 marks: 2, 3, 3, 3]

An eagle flies with a constant velocity of < 2, 4, 1 > metres per minute. At 7 am, the eagle flies past the point A with position vector < 10, –10, 2 > . Find:

(a) the position vector of the eagle at 7.15 am (same day)

(b) when the eagle is 50 m from A

(c) when and where the eagle crosses the x-z plane.

(d) using a vector method, the closest distance between the eagle and the point B with position vector < 20, 5, 8 > m.

Calculator Assumed

3. [11 marks: 2, 3, 3, 3]

At 0900 hours, the position and velocity vectors of particles P, Q and R are respectively $< 0, -1, -2 >$ m , $< 1, 2, 2 >$ ms^{-1} ; $< 4, 1, 6 >$ m , $< -1, \alpha, -2 >$ ms^{-1} and $< -4, 1, -2 >$ m and $< 3, 1, \beta >$ ms^{-1} .

(a) Find the position vectors of P, Q and R, t seconds after 0900 hours.

(b) If these velocities were maintained, all three particles will collide simultaneously. Use vector methods to find the value(s) of α and β.

(c) If these velocities were maintained, only P and Q will collide. Use vector methods to find the value(s) of α and β.

(d) If these velocities were maintained, only two of the three particles will collide. Use vector methods to find the value(s) of α and β.

Calculator Assumed

4. [13 marks: 3, 4, 2, 4]

The position vectors of objects P and Q at time t seconds is given by
$\mathbf{OP}(t) = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ metres and $\mathbf{OQ}(t) = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} -1 \\ -1 \\ 3 \end{pmatrix}$ metres respectively.

(a) Show that P and Q do not collide.

(b) Calculate the closest distance between P and Q.

(c) State the vector equations of the paths of P and Q.

(d) Calculate the closest distance between the paths of P and Q.

Calculator Assumed

5. [10 marks: 5, 5]

Particle A leaves the point $< 1, a, 2 >$ m with constant velocity $< -1, 2, 5 >$ ms^{-1}. Simultaneously, particle B leaves the point $< -3, 1, 10 >$ m with constant velocity $< 1, 1, u >$ ms^{-1}. a and u are real constants.

(a) For $a = -1$, determine the value(s) of u for which the particles do not collide.

(b) For $a = -4$, determine the value(s) of u for which the paths of A and B will intersect.

---

6. [14 marks, 3, 4, 4, 3]

Particles A and B start moving at 0800 hours. The position vectors of A and B t hours after 0800 hours are respectively $\boldsymbol{r}_A = \begin{pmatrix} 10+4t \\ -20+2t \\ 4-t \end{pmatrix}$ and $\boldsymbol{r}_B = \begin{pmatrix} 20+t \\ 20-3t \\ -11+t \end{pmatrix}$.

(a) Prove that A and B will not collide.

Calculator Assumed

6. (b) The paths traced by A and B respectively meet at the point P. Determine the position vector of the point P.

(c) Determine with reasons, which of the two particles A or B is the first to reach P. State the exact distance between the two particles at the time the first particle reaches P.

(d) Determine the vector equation of the plane of motion of A and B.

Calculator Assumed

7. [8 marks: 5, 3]

A drone located at P is to be flown to Q. The position vector of Q relative to P is $< 100, 80, 40 >$ m. A wind is blowing with constant velocity $< 0.8, 1.3, 0.4 >$ ms^{-1}.

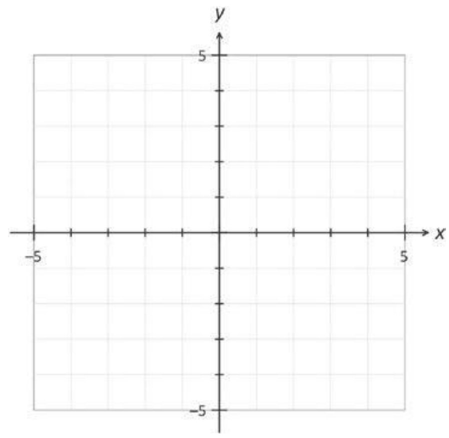
(a) Determine $\boldsymbol{v}$ the velocity of the drone if it is flown from P to Q at its maximum speed of 1.3 ms^{-1}.

(b) Comment briefly on the significance of your answer in (a).

Calculator Free

8. [9 marks: 3, 4, 2]

The position vector of a particle P at time t seconds is given by
$\boldsymbol{r}(t) = < 1 + 2\sin(\pi t), 2 - 3\cos(\pi t) >$ cm where $0 \le t \le 2$.



(a) Determine the Cartesian equation of the path of P.

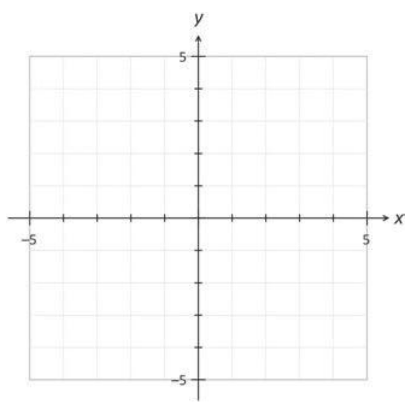
(b) On the axes provided, sketch the graph of the path of P. Indicate the direction of motion of P.

(c) Determine the shortest distance the particle gets to point with position vector $< 1, 2 >$.

---

9. [6 marks: 4, 2]

The position vector of a particle P at time t seconds is given by
$\boldsymbol{r}(t) = < 1 + \tan^2 t, 2 - \sec^2 t >$ cm where $t \ge 0$.



(a) Determine the Cartesian equation of the path of P.

(b) On the axes provided, sketch the graph of the path of P.

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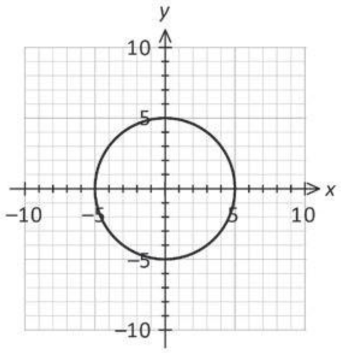
Calculator Free

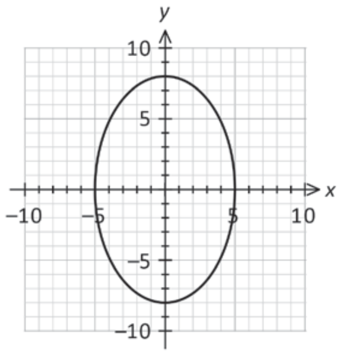
10. [12 marks: 3, 3, 3, 3]

The position vector a particle at time t seconds ($t \geq 0$) is given by $\boldsymbol{r} = < a\cos(\alpha t), b\sin(\beta t) >$. For each of the given diagrams, provide one possible set of values of the constants a, b, α and β.

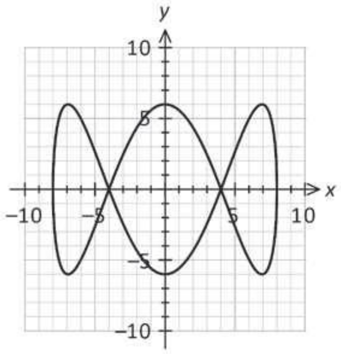
(a)



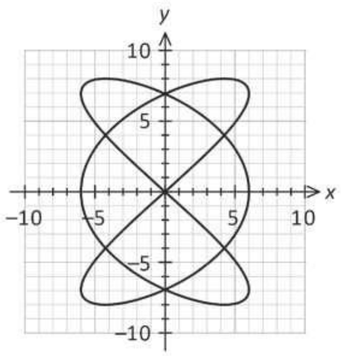
(b)



(c)

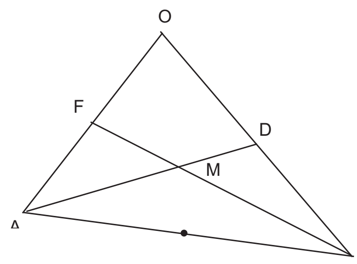


(d)



Calculator Assumed

1. [13 marks: 2, 2, 5, 3, 1]



OAB is a triangle with **OA** = ***a*** and **OB** = ***b***. D, E and F are the midpoints of OB, AB, and OA respectively. **AM** = α**AD** and **MF** = β**BF**.

(a) Find **AD** and **BF** in terms of ***a*** and ***b***.

(b) Find **AM** and **MF** in terms of ***a*** and ***b***.

(c) Use your answers in (b) to find α and β.

(d) Show that **OM** = μ**OE** giving the value of μ .

(e) Comment on the significance of the location of M in terms of the lines OE, AD and BF.

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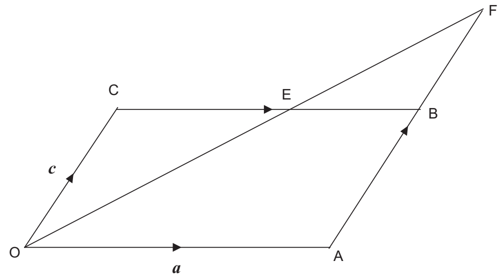
Calculator Assumed

2. [8 marks]



OABC is a parallelogram. The point E divides CB in the ratio α : β. That is, the point E is such that $\mathbf{EB} = \dfrac{\beta}{\alpha+\beta}\mathbf{CB}$. OE extended meets the AB extended at F. Use vector methods to prove that:

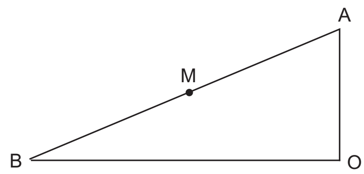
Area of ΔFEB = $\left(\dfrac{\beta}{\alpha+\beta}\right)^2$ × Area of ΔFOA.

[Hint: Let $\mathbf{EF} = \lambda\mathbf{OF}$ and $\mathbf{BF} = \mu\mathbf{AF}$.]

Calculator Assumed

3. [8 marks: 1, 3, 4]

OAB is a right angled triangle with $\angle$ AOB = 90°. $\mathbf{OA} = \boldsymbol{a}$ and $\mathbf{OB} = \boldsymbol{b}$.
M is the midpoint of AB.



(a) Explain why $\boldsymbol{a} \cdot \boldsymbol{b} = 0$.

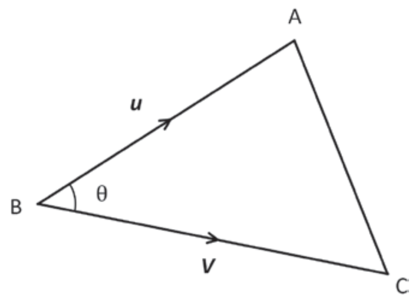
(b) Find $|\mathbf{BM}|^2$ in terms of a and b, where $|\boldsymbol{a}| = a$ and $|\boldsymbol{b}| = b$.

(c) Hence, prove that M is the centre of a circle passing through A, B and O.

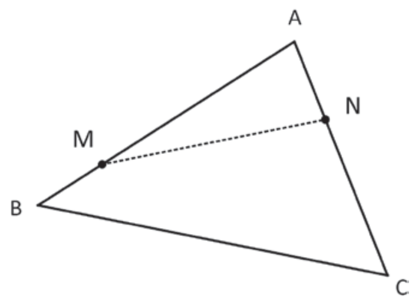
Calculator Assumed

4. [12 marks: 2, 5, 5]

(a) The given diagram shows triangle ABC formed by the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ where θ is the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$. Use the result $|\boldsymbol{u} \times \boldsymbol{v}| = |\boldsymbol{u}|\,|\boldsymbol{v}|\sin\theta$ to prove that the area of this triangle is $\dfrac{1}{2}|\boldsymbol{u} \times \boldsymbol{v}|$.



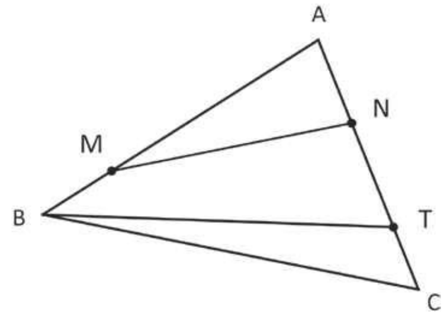
The points A, B and C respectively with position vectors $2\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}$, $7\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$ and $5\boldsymbol{i} + \boldsymbol{j} - 4\boldsymbol{k}$ form the vertices of a triangle. The point M on AB divides AB in the ratio 4 : 1, while the point N on AC divides AC in the ratio 1 : 2.



(b) Calculate the exact area of the quadrilateral MNCB.

Calculator Assumed

4. (c) The point T lies on AC such that the area of the quadrilateral MNTB is $9\sqrt{2}$. Determine the position vector of the point T.



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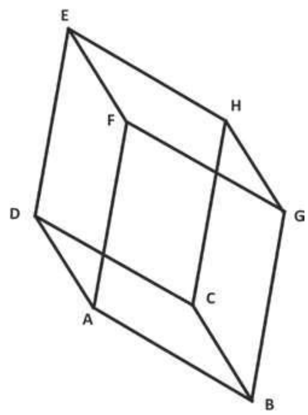
5. [5 marks]

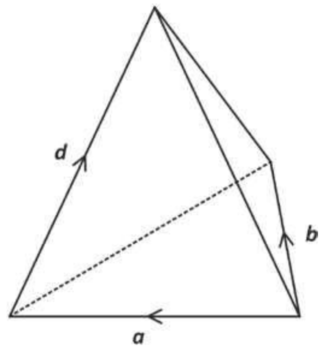
The accompanying diagram shows a parallelepiped. The equations of the faces ABCD and EFGH are respectively $x + 7y - 5z = 23$ and $x + 7y - 5z = 38$. Prove that the perpendicular distance between the faces ABCD and EFGH is $\sqrt{3}$.



Calculator Assumed

6. [7 marks: 2, 5]

The accompanying diagram shows a tetrahedron. Two of the adjacent edges of the tetrahedron are represented by the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. A third edge is represented by the vector $\boldsymbol{d}$.



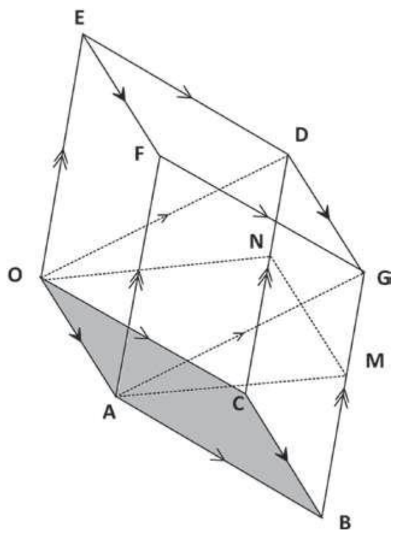
(a) Prove that the area of the face bounded by $\boldsymbol{a}$ and $\boldsymbol{b}$ is given by $A = \frac{1}{2}|\boldsymbol{a} \times \boldsymbol{b}|$.

(b) Prove that the volume of the tetrahedron is given by $V = \frac{1}{6}|\boldsymbol{d} \bullet (\boldsymbol{a} \times \boldsymbol{b})|$.

---

7. [7 marks: 3, 4]

The given diagram shows a parallelepiped OABCDEFG. The opposite faces of the solid are congruent parallelograms which are parallel to each other. **OA** = $\boldsymbol{a}$, **OC** = $\boldsymbol{c}$ and **OE** = $\boldsymbol{e}$. The point M is such that **BM** = α**BG**. The point N is such that **CN** = α**CD**.



(a) Prove that **NM** = $\boldsymbol{a}$.

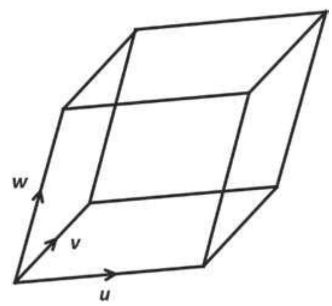
Calculator Assumed

7. (b) The volume of parallelepiped OABCDEFG is given by $|(\boldsymbol{a} \times \boldsymbol{c}) \bullet \boldsymbol{e}|$.
Use this result to prove that $\alpha = \frac{3}{4}$ given that the ratio of the volume of ODGABC to the volume of ONMABC is 4 : 3.

---

8. [6 marks]

The edges of a parallelepiped are parallel and congruent to the vectors $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ as shown in the accompanying diagram. Determine the volume V of the parallelepiped. Hence, deduce that:
$|\boldsymbol{w} \bullet (\boldsymbol{u} \times \boldsymbol{v})| = |\boldsymbol{u} \bullet (\boldsymbol{v} \times \boldsymbol{w})| = |\boldsymbol{v} \bullet (\boldsymbol{w} \times \boldsymbol{u})|$.



Calculator Assumed

9. [6 marks]

The accompanying diagram shows a rectangular prism with a section removed. O is the origin of the *x*-*y*-*z* axes. The position vectors of vertices A, C and E are $\boldsymbol{a}$, $\boldsymbol{c}$ and $\boldsymbol{e}$ respectively. Prove that the volume of this solid is given by $V = \frac{5}{6}\left|\boldsymbol{e} \bullet (\boldsymbol{a} \times \boldsymbol{c})\right|$.